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PAUL KUNITZSCH/RICHARD LORCH

Maslama's Notes on Ptolemy's *Planisphaerium* and Related Texts

Vorgelegt von Herrn Paul Kunitzsch
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Contents

Introduction

Of Ptolemy's treatise on the projection of the circles on the sphere onto a plane, commonly known as the *Planisphaerium*[1], the original Greek text seems to be lost.

Around, or before, AD 900 the *Planisphaerium* was translated into Arabic by an unknown translator[2] in the Islamic East, probably in Baghdad. The Arabic version was used, and the introductory phrases cited *verbatim*, by Ibrāhīm ibn Sinān ibn Thābit (909–946), the grandson of Thābit ibn Qurra (d. 901), in his treatise on the astrolabe[3]. The Arabic text of the *Planisphaerium* has survived in three manuscripts[4], of which one (in Kabul) has remained inaccessible until now; one (Istanbul, Aya Sofya 2671, 76v–97r, dated 621 H = AD 1224) has been known for some years and was edited in facsimile[5]; and of the third (Tehran, private library of Khān Malik Sāsānī, dated 607 or 617 H = AD 1210 or 1220-21) copies became

[1]Cf. van der Waerden, col. 1829–31; Toomer, pp. 197f., 205.

[2]The major bio-bibliographical source of early Arabic-Islamic science, Ibn al-Nadīm's *Fihrist* (completed in 987–88), does not mention the *Planisphaerium* and its translation. On the other hand, it registers Pappus' commentary on the *Planisphaerium* translated into Arabic by Thābit ibn Qurra (ed. Flügel, p. 269,8–9; the Arabic text has not yet been found in manuscripts).

[3]Cf. Kunitzsch [II] and [III].

[4]Cf. Sezgin, V (1974), p. 170.

[5]The edition is in Anagnostakis.

available to us recently. The two manuscripts of Istanbul and Tehran represent the same recension of the text, but Tehran has many better readings; further, Tehran contains all the diagrams (though sometimes rather carelessly drawn), whereas Istanbul had left their places blank[6].

In the course of the tenth century the *Planisphaerium* was received in Muslim Spain, where Maslama al-Majrīṭī studied and commented on it (see below). In 1143, in Tolosa, Hermann of Carinthia — known for his very free translations from Arabic into Latin — translated the Arabic text into Latin[7]. Probably induced by the comments added to the text distinctly under Maslama's name, Hermann erroneously thought that the text of the *Planisphaerium* itself had been translated into Arabic by Maslama; this wrong assumption has been retained by most modern scholars until recently[8]. But it is now clear — especially because of Ibrāhīm ibn Sinān's quotations — that the Arabic text of the *Planisphaerium* existed in the Arabic East half a century or more before Maslama.

The Spanish-Arabic astronomer and mathematician Maslama al-Majrīṭī (d. 398 H = AD 1007–08), highly praised as "the *imām* of mathematicians in al-Andalus at his time and more learned than his predecessors in the science of the heavens and the motions of the planets"[9], is well-known for his revision of al-Khwārizmī's astronomical tables[10]; he is also

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[6]For details and a list of better readings from MS Tehran, see Kunitzsch [III]. Cf. also the table, below on p. 97.

[7]Edited by Heiberg, in Ptolemaeus, *Opera*, vol. II; German translation by Drecker. For the identification of Tolosa (in Spain, south of San Sebastián), cf. Lemay, p. 451. An Italian translation (from the text of Commandinus and two other Renaissance printed editions) was made by Sinisgalli and Vastola.

[8]To the places cited in Kunitzsch [II], footnote 10, could be added Burnett, p. 108, and Neugebauer [II], p. 871.

[9]Ṣāʿid al-Andalusī, p. 168,ult.–169,2; Blachère, p. 129.

[10]Not extant in Arabic; Latin translation by Adelard of Bath (c.

said to have studied the *Almagest* intensively[11]. Among the works that reached him from the Arabic East there was also the *Planisphaerium*. He studied it and wrote a number of notes and an additional extra-chapter to it. These notes, eleven in number, and the Extra-Chapter together with some "Chapters indispensable for whosoever wants to construct an astrolabe" are assembled and transmitted in one block, in MS Paris, BN ar. 4821 (written in Iran, dated Sha'bān 544 = December 1149)[12].

Maslama's notes were also translated into Latin and transmitted — throughout citing Maslama's name — in the Latin manuscripts, partly incorporated into the text of the *Planisphaerium* and partly in the margins. Heiberg in his *Prolegomena* gave a confused and incomplete survey of them. An investigation of the Latin sources (MSS **A**, **B**, **C**, **D**, **E**, **F** and **X**, the printed edition of Commandinus, 1558, already used by Heiberg, and MSS **K**, **L**, **M**, **N**, **W**, **S**, **P** added by us) now allows a much more detailed classification of the Latin material.

Our results concerning the *Planisphaerium* and Maslama's notes, in Arabic and Latin, can be summed up as follows.

Of the Arabic text of the *Planisphaerium* there seem to have existed two recensions, an "Eastern" recension represented by the Arabic manuscripts Istanbul (**Y**) and Tehran (**T**), and a "Western" recension represented by Hermann's Latin translation and by some of the quotations in Maslama's notes (in Arabic)[13].

1126), ed. by Suter, Björnbo and Besthorn; English translation and comments in Neugebauer [II].

[11]Ṣā'id al-Andalusī, p. 169,2–3; Blachère, p. 129.

[12]First signalled by Vajda, pp. 7–9. Later, the Extra-Chapter to the *Planisphaerium* and the Astrolabe Chapters were edited, with a Spanish translation, by J. Vernet and M.A. Catalá. The notes proper to the *Planisphaerium* have hitherto remained unedited.

[13]In most of the notes the Arabic compiler or redactor of the

Hermann's translation follows the "Western" recension of the Arabic text of the *Planisphaerium* and therefore is often different from the text transmitted in **Y** and **T** ("Eastern" recension).

Maslama's notes to the *Planisphaerium*, eleven in number, exist in Arabic, together with the Extra-Chapter and the Astrolabe Chapters, in MS Paris, BN ar. 4821. The notes in the form given here mostly contain quotations from the text of the *Planisphaerium*, some of them in the "Western" recension.

The Latin material falls into three groups, each of which is apparently due to a different translator. Group I comprises seven of Maslama's notes appearing directly in the text of the *Planisphaerium*, in all manuscripts (notes 1, 2, 3, 6, 7, 9 [all but the beginning] and 11). Group II, by another translator, comprises notes 5, 8, 9 (beginning only) and 10, transmitted variously in the margins, within or at the end of the *Planisphaerium*, in MSS **ABNP** only. Group III, by a third translator, comprises notes 4, 5, 7 (part only), 8, 9 (beginning only) and 10, transmitted in the margins of MSS **EF** alone. Further, each of the translators of **ABNP** and **EF** also translated Maslama's Extra-Chapter. Finally, the translator of **EF** also translated Maslama's Astrolabe Chapters. In addition, the translator of **EF** translated portions of the *Planisphaerium* itself from the "Eastern" recension, parallel to Hermann's version, which was made from the "Western" Arabic recension, and several terms or short phrases by him were written over

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collection in MS Paris 4821 quotes the beginning and the ending words of the proposition of the *Planisphaerium* to which each note was added (cf. the apparatus of the Arabic text, below; in the Latin rendering this section is never present). It should be noted that Maslama's counting of the propositions of the *Planisphaerium* as visible in his notes, and also the proposition-counting in some of the Latin manuscripts, differ from Heiberg's section numbers; cf. the table of correspondence added to the Comments, below on p. 97.

the corresponding words or in the margins of Hermann's text.

The translator of Group I seems to be Hermann. It is clear from his own words in the Introduction that in his Arabic text of the *Planisphaerium* he saw Maslama's notes; no doubt it was for this reason that he assumed that Maslama was the Arabic translator of the *Planisphaerium* itself. The style of the notes of this group corresponds to Hermann's style of translating, mostly very free and only rarely close to the original. The (unknown) translator of Group II offers a rather literal translation, different from Hermann's style. The translator of Group III (again unkown) also translated rather literally, but not as literally as translator II. His terminology is often different from that of translator II; e.g. Arabic *ṣafīḥa* ("plate") is *planisperium* in I, *tabula* in II, *lamina* in III; *mas'ala* ("question, problem, proposition") is *questio* in II, *propositio* in III; *maṭāliʿ* ("ascension(s)") is *ortus* in I, *ascensio(nes)* in II, *elevatio(nes)* in III. In the diagram letters, Hermann's usage in the *Planisphaerium*[14] is followed in Groups I and II, especially Ar. *sīn* = *C*, *ʿayn* = *Y* and *wāw* = *O*. In III there is also *sīn* = *C*, but *ʿayn* = *O* and *wāw* = *V* (except for note 9 [beginning], where III also has *ʿayn* = *Y* and *wāw* = *O*). From this, especially the use of *C* for *sīn*, one would be inclined to deduce that translators II and III belonged to the *équipe* of Hermann or were at work after him (other translators such as Adelard of Bath and Gerard of Cremona used *S* for *sīn*, cf. the tables in Kunitzsch [I]).

Below, the Arabic text of Maslama's notes together with the various Latin versions and some additional texts are edited. The Arabic text is accompanied by an English translation. A section with comments follows.

The extra translations by translator III from the "Eastern" recension of the *Planisphaerium* are edited at the end as

[14]Cf. Kunitzsch [I], p. 8 note 7; Tables 2, 3, 4. See also footnote 2 in the Comments, below.

Appendix I. Further, in Appendix II is added the edition of the so-called *Propositiones planisperii*, sixteen in number (one for each proposition of the *Planisphaerium* according to the medieval counting), and of two further, untitled, small texts concerning the *Planisphaerium*. In the *Propositiones* an unknown Latin scholar gave a summary of the *Planisphaerium*, perhaps for didactic purposes, and in the two short pieces of text mathematical topics occurring in Ptolemy's work are discussed. In *Prop*. I, XII and XIII Maslama's notes no. 1, 6 and 7 are also reported (note 1 in a much expanded form).

It is not possible at this stage to fix exactly the number and identity of the several individuals involved in the creation of all these translations and texts centred around the *Planisphaerium*. Only Hermann of Carinthia can be affirmed as a translator: of the *Planisphaerium* itself and of the notes of Group I, i.e. the "internal" notes. It looks as if the Arabic manuscript used by him did not contain the other notes and the extra material. The translators of Groups II and III worked independently of each other, each of them having to hand Hermann's Latin version and other Arabic manuscripts, from which they translated the notes and the extra material absent from Hermann's version and from which they provided alternative translations of short passages of text. In addition there remains the problem of the authorship of the comments to the Astrolabe Chapters, of the *Propositiones* and of the two short texts transmitted together with them. For all these no names can yet be put forward with any certainty.

At the end, one conclusion at least is obvious: that the *Planisphaerium* found more interest and provoked greater reaction in medieval Europe than in the Islamic world.

Edition of the texts

1. Maslama's notes, Arabic text

MS Paris, BN ar. 4821, 69v–75v

<69v> بسم الله الرحمن الرحيم

تعاليق لمسلمة بن أحمد الأندلسى
على كتاب بطلميوس فى تسطيح بسيط الكرة

<١> قال فى الشكل الأول عند قول بطلميوس »أعنى أن تجعل أوائل البروج على النقط التى عليها تقسم الدوائر الموازية لمعدل النهار التى ترسم بالطريق الذى أوضحنا على البعد الموافق لبعد كل واحد من البروج من دائرة معدل النهار فى الكرة المجسمة دائرة فلك البروج«، قال أراد بطلميوس بقوله هذا أنه تعمل دوائر موازية لدائرة معدل النهار التى هى دائرة ابجد ببعد ميل برج برج فيما بينها وبين دائرة طل التى هى المنقلب الصيفى وفيما بينها وبين

English translation

[69v] In the name of God, the Merciful, the Compassionate

Comments by Maslama ibn Aḥmad al-Andalusī on Ptolemy's Book on the Projection of the Sphere[1]

[1] He [i.e. Maslama] said about the first proposition[2], where Ptolemy said, "i.e. the beginnings of the signs are put on the points on which the circles parallel to the equator — which are drawn in the way that we have explained, in the distance corresponding to the distance of each of the signs from the equator circle on the solid sphere — intersect the circle of the zodiac"[3], [here] he said: with this statement Ptolemy intended that the circles parallel to the equator circle, which is circle $ABGD$, are constructed with the distance of the declination of each sign between it [*sc.* the equator] and circle TL, which is the summer tropic, or between it and

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[1] Ar. *tasṭīḥ basīṭ al-kura*, an almost literal rendering of the Greek title of the *Planisphaerium* cited in the *Suda*, ἅπλωσις ἐπιφανείας σφαίρας. The true reading of the manuscript is *basīṭ* (with *yā'*), not *basṭ* as cited in Vernet-Catalá, pp. 18 and 20. Therefore all that is said in Kunitzsch [4], pp. 517f., no. 3 (based on Vernet-Catalá) about *basṭ al-kura* must be changed to the correct spelling *basīṭ al-kura*. In this form, however, it appears less clear that it was the Arabic original of the Latinized title *waztalcora* (and variants).

[2] I.e. in the medieval counting; cf. the table below on p. 97.

[3] For the identification of these quotations in the Arabic of the *Planisphaerium* (MSS **Y** and **T**), see the apparatus to the Arabic text, below.

دائرة كم التى هى المنقلب الشتوى فحيث ما قاطعت هذه الدوائر الموازية لمعدل النهار التى داخلها وخارجها دائرة فلك البروج فتلك النقط تكون أوائل البروج وكذلك لو عملت دوائر موازية لبعد ميل درجة درجة من فلك البروج لقسمت دائرة فلك البروج على جزءٍ جزءٍ ولكنك إذا أردت أن تخرج هذه الصناعة إلى الفعل حدث لك فى ذلك تقريب كثير إذ الدوائر المتماسة لا تتماس بالعقل إلا على نقطة وبالفعل على خط .

<٢> قال مسلمة بعد تمام الشكل الثالث لو <70r> قصد بطلميوس إلى البرهان بالخلف لكان أقرب إلى الفهم وذلك أنه حيث قال »أنا نصل خط هح ونخرجه على استقامة حتى يقطع دائرة الأفق على نقطة ط« لو قال فإن لم يمر بنقطة ط فليمر بنقطة أخرى إما دونها وإما فوقها فليمر أولاً بنقطة م وننفذه إلى دائرة فلك البروج حتى يقاطعها على نقطة ز فيكون ضرب حه فى هم كضرب اه فى هج وضرب اه فى هج كضرب به فى هد فضرب به فى هد إذاً كضرب حه فى هم وضرب به فى هد كضرب حه فى هز فيجب إذاً أن يكون ضرب حه فى هم كضرب حه فى هز فهم إذاً مساوٍ لهز هذا خلف لا يمكن وكذلك لا يمكن أن يخرج على استقامة فوق نقطة ط .

<٣> قال مسلمة بعد فراغ جميع الكلام فى الشكل الرابع، ولما استبان أن الدوائر الموازية لمعدل النهار بأى بعد كانت يحدّها خط دز ودكح وأن نصف قطر الدائرة الموازية لمعدل النهار من ناحية الجنوب يكون من نقطة ه إلى

circle KM, which is the winter tropic. Wherever these circles parallel to the equator, inside it and outside it, cut the zodiac circle, these points are the beginnings of the signs. Similarly, if you drew parallel circles for the distance of the declination of each degree of the zodiac, they would divide the zodiac circle at each degree. But when you want to make this construction in practice, great approximation takes place for you, since mutually tangent circles are only tangent at a point in the mind, but on a line in practice.

[2] Maslama said after the end of the third proposition: if [70r] Ptolemy had intended a proof by contradiction [i.e. *reductio ad absurdum*], it would have been easier to understand. That is because where he said "we join line EH and produce it in a straight line until it cuts the horizon circle at point T" [it would have been easier to understand] if he had said [the following]: if it does not pass through point T, let it pass through another point, either below it or above it; and first let it pass through M. We extend it to the zodiac circle, so that it intersects it at point Z. So $HE \cdot EM = AE \cdot EG$ and $AE \cdot EG = BE \cdot ED$. Therefore $BE \cdot ED = HE \cdot EM$. But $BE \cdot ED = HE \cdot EZ$. It is therefore necessary that $HE \cdot EM = HE \cdot EZ$: EM is therefore equal to EZ. This is a contradiction, not possible. Similarly it is impossible that it is produced in a straight line above point T.

[3] Maslama said after finishing the whole text about the fourth proposition: since it has become clear that the circles parallel to the equator, at whatever distance they are, are defined by line DZ and DKH and the semidiameter of the circle parallel to the equator on the south side is from point E to

النقطة التى عليها يتقاطع خط دط مع خط هج وفرضنا قوس جط تسعة وثمنين جزءًا وجب أن يقع التقاطع على نصف قطر دائرة بعدها من دائرة معدل النهار إلى ناحية الجنوب تسعة وثمنون جزءًا وقد علمنا أن بعد القطب الجنوبى من دائرة معدل النهار تسعون جزءًا وهو ميل كل قوس دج فإنما يجب أن يحدّ القطب الجنوبى فى هذا <70v> السطح تقاطع خط هج مع خط موازٍ له يخرج من نقطة د والمتوازيان لا يتقاطعان فالقطب الجنوبى إذاً لا يمكن وضعه فى هذا السطح وكذلك لو وضعنا نقطة ه القطب الجنوبى لكان القطب الشمالى لا يمكن وضعه معه فى السطح وأيضاً قد أثبت عندنا بطلميوس أن الخطوط المستقيمة المارة على مركز ه هى بدل الدوائر المعروفة بدوائر نصف النهار المارة على القطبين وأن تلك الدوائر تتقاطع فى موضعين فالموضع الواحد القطب الشمالى والآخر الجنوبى وهذه الخطوط المستقيمة لا تتقاطع إلا فى موضع واحد فإذا فرضنا ذلك الموضع القطب الواحد لم يكن إلى حيث نفرض الثانى إذ الخطوط المستقيمة لا تتقاطع فى موضعين وهذا استبان ما قاله بطلميوس فى صدر هذا الكتاب .

<٤> قال مسلمة فى الشكل الخامس وهو الذى يقول بطلميوس فى أوله »فإذ قد وصفنا ذلك فنبيّن فى مثل هذه الصورة أنه ترى مقادير المطالع وجميع ما يعرض فيها على ما بيّنّا فى الكرة المجسمة« بعد قوله »فزاوية هطف إذاً معلومة، وذلك ما أردنا أن نبين«، وكان له أيضاً طريق أسهل من ذلك وذلك أنه قد ذكر قطر كل واحدة من الدائرتين الموازيتين لمعدل النهار ببعد واحد

the point at which line DT and line EG intersect, we suppose arc GT to be 89 degrees; the intersection must fall on the semidiameter of a circle whose distance from the equator towards the south is 89 degrees. We know that the distance of the south pole from the equator circle is 90 degrees — it is the declination of the whole arc DG. But the south pole must be defined in this [70v] plane as the intersection of line EG with a line parallel to it issuing from point D. But parallel lines do not intersect: it is therefore impossible to place the south pole in this plane. Similarly, if we had supposed E to be the south pole, it would have been impossible to place the north pole with it in the plane. In our opinion it has been established by Ptolemy that the straight lines passing through centre E take the place of the circles known as the meridian circles, which pass through the two poles, and that these circles intersect in two places: the one place is the north pole and the other the south. But these straight lines intersect in only one place. When we suppose that place to be the one pole, there is nowhere we can suppose the second [to be], since straight lines do not intersect in two places. This [makes] clear what [Ptolemy] said at the beginning of this book.

[4] Maslama said about the fifth proposition, which is the one at the beginning of which [Ptolemy] said, "Now that we have described that, we shall show with a similar diagram that the quantities of the ascensions and everything that appears together with them is seen just as we proved in the solid sphere", after his [i.e. Ptolemy's] words "and so angle ETF is therefore known. Q.E.D.": there is also a way for this easier than that. That is that he mentioned the diameter of each of the two circles parallel to the equator at the same distance

إلى الجنوب وإلى الشمال فإذا كان خط هك نصف قطر الدائرة الجنوبية من معدل النهار فخط هن نصف قطر الدائرة الشمالية التى هى <71r> بذلك البعد وقد تقدم له ذكر كمية قطرها فى المسئلة التى قبل هذه والبرهان على ذلك فى المسئلة الثانية من كتابه .

<ه> قال مسلمة فى الشكل وهو الذى فى أوله »ولكن نجعل الصورة على حسب مقدار الموضع المعلوم الذى نريد أن نرسم فيه ما ذكرنا وحتى يتهيأ لنا أن نرسم مواضع الكواكب الثابتة إن أردنا ذلك« بعد قوله »شبيهة بقوس جز، وذلك ما أردنا أن نبين«، ولو قصد بطلميوس فى هذه المسئلة إلى ما أذكره لكان أقرب مأخذاً إن شاء الله وذلك أنه إذا وصل د بنقطة ز لو أخرج من نقطة ج خطاً موازياً لخط دز وهو جم ثم عمل الدائرة التى تكون مدار الحمل ببعد هم لكانت دائرة الحمل وذلك أن زاوية نمه مساوية لزاوية زده فقوس بجز شبيهة بقوس سلن فتبقى قوس نل شبيهة بقوس جز .

(Figure 1)

وله وجه آخر قريب أيضاً وذلك أنّا نجعل أعظم الدوائر الواقعة فى الآلة دائرة ابجد حول مركز ه ونخرج قطرى الدائرة يتقاطعان على زوايا قائمة ونريد أن نرسم داخلها <دائرة> يكون بعد هذه الدائرة من المرسومة داخلها ببعد قوس شبيهة بقوس جز فنصل بز وليقاطع <71v> خط هج على نقطة ح ونجعل ه مركزاً ونرسم دائرة ببعد هح وهى دائرة حكط فأقول إن قوس كط شبيهة بقوس زج، برهان ذلك أنا نصل جب حط فحط موازٍ لجب فزاوية كحط مساوية

to the south and to the north. When line EK is the semidiameter of the circle south of the equator, line EN is the semidiameter of the northern circle that is [71r] at that distance. He had already mentioned the amount of its diameter in the question that precedes this one and the proof of that is in the second question in his book.

[5] Maslama said about the [...] proposition, which is the one that begins "But we make the diagram according to the quantity of the known position, in which we want to draw what we mentioned, and so that it is possible for us to draw the positions of the fixed stars, if we want that", after his [i.e. Ptolemy's] words "similar to arc GZ. Q.E.D.": if Ptolemy had intended in this question what I am going to say, it would have been easier to grasp, God willing. That is that when he joined D to point Z, if from point G he had drawn a line parallel to line DZ, which is GM, and had then constructed the circle, which is the course of Aries, with distance EM, it would have been the circle of Aries. That is because angle NME is equal to angle ZDE: so arc BGZ is similar to arc CLN, and then arc NL is similar to arc GZ.

[Fig. 1]

There is also another easy way. That is that we make the greatest of the circles occurring on the instrument circle $ABGD$ about centre E and draw the two diameters of the circle intersecting at right angles. Inside it we want to draw a circle so that the distance of this circle [*sc.* $ABGD$] from the one drawn inside it is in the distance of an arc similar to arc GZ. We join BZ; let it intersect [71v] line EG at point H. We make E a centre and draw a circle with distance EH, which is circle HKT. *I say*: arc KT is similar to arc ZG. *Proof*: we join GB, HT. So HT is parallel to GB; and therefore angle KHT is equal

لزاوية جبز وكل واحدة منهما على محيط الدائرة فقوس كط شبيهة بقوس جز فإذا كانت دائرة حكط معدل النهار كانت دائرة ابجد مرسومة عنها ببعد كط التى هى شبيهة بقوس جز وذلك ما أردنا بيانه .

(Figure 2)

<٦> قال مسلمة فى الشكل وهو الذى أوله »وأيضاً ينبغى أن نتمم غرضنا بأن نبين كيف ترسم الدوائر التى حالها عند الدائرة التى تمر بوسط البروج كحال الدوائر التى تقدم ذكرها عند معدل النهار« عند قوله »فإنها تقطع الدائرة الأخرى الباقية على القطر أيضاً«، وهذه الدوائر إذا رُسمت فى البسيط المسطح ومرت بدرجة كوكب من الكواكب الثابتة فإنها تمر بالكوكب بعينه وكذلك إن مرت بالكوكب بعينه فإنها تمر بدرجته والخطوط المستقيمة التى تمر بمركز دائرة معدل النهار فى السطح إذا مرت بالكوكب بعينه فإنها تمر بالدرجة التى معها يتوسط الكوكب السماء وإن مرت الخطوط بالدرجة التى معها يتوسط الكوكب السماء فإنها تمر <72r> بالكوكب بعينه فاعلم .

<٧> قال مسلمة فى الشكل الذى أوله »وقد يمكننا أن نضع فى الصفيحة الدوائر الموازية لدائرة فلك البروج أيضاً على هذا المثال« وفى آخره »فنقط ن ع م ف على محيط دائرة«، فإذا عملت دائرة موازية لدائرة فلك البروج يكون ميلها عن فلك البروج بمقدار عرض كوكب من الكواكب الثابتة ثم طرحت قوساً تمر بقطب فلك البروج الذى هو نقطة ك فى المسئلة التى قبل هذه

to angle GBZ. Each of them is on the circumference of a circle: therefore arc KT is similar to arc GZ. When circle HKT is the equator, circle $ABGD$ is drawn at a distance KT from it, which is similar to arc GZ. Q.E.D.

[Fig. 2]

[6] Maslama said about the [...] proposition, which is the one that begins "Also we must complete our aim by showing how to draw the circles whose situation in relation to the circle passing through the middle of the signs is as the situation of the previously mentioned circles in relation to the equator", at his [i.e. Ptolemy's] words "they cut the other, remaining, circle on the diameter also": when these circles are drawn on the flat surface and [one of them] passes through the degree of one of the fixed stars, it passes through the star itself; and, similarly, if it passes through the star itself, it passes through its degree. When the straight lines that pass through the centre of the equator circle in the plane pass through the star itself, they pass through the degree with which the star mediates; and if the lines pass through the degree with which the star mediates, they pass [72r] through the star itself. Understand it well!

[7] Maslama said about the proposition that begins "It is also possible for us to put the circles parallel to the zodiac on the plate according to this model" and ends "Therefore points N, Y, M, F are on the circumference of a circle": when you construct a circle parallel to the zodiac circle so that its declination from the zodiac is in the quantity of the latitude of one of the fixed stars and then drop an arc passing through the pole of the zodiac, which is point K in the question that precedes this one,

وبدرجة الكوكب فى فلك البروج وتقطع فلك البروج بنصفين فإنها ستقطع فلك معدل النهار أيضاً بنصفين فحيث ما تقاطعت هذه القوس مع الدائرة الموازية لفلك البروج فتلك النقطة موضع الكوكب فى الصفيحة ومن هذه الصورة تعمل الدوائر الموازية لدائرة الأفق وهى المقنطرات ويقوم عليها البرهان منها .

<٨> قال مسلمة فى الشكل الذى أوله »وينبغى أن نبين أن مراكز الدوائر الموازية لدائرة البروج التى ترسم على هذا المثال تكون مختلفة أبداً« وفى آخره »فليس نقطة و لمركز الدائرة تمر بنقطتى نس، وذلك ما أردنا أن نبين«، لم يبين أن المراكز ليست على نقطة واحدة إلا بعد أن بيّن أن خط مس أطول من خط نل وقد نبين ذلك بأقرب من هذا وذلك أن نعيد الشكل وذلك أن نخرج من نقطة ن خطاً يكون عموداً <72v> على خط دل وهو نع ونبين أن دن أقصر من دس ودل أقصر من دم فلنقطع فى خط دس خطاً مساوياً لخط دن وهو دط ومن خط دم خطاً مساوياً لدل وهو دز فيصير مثلث دطز مساوياً لمثلث دنل ويقع عمود نع كوقوع عمود طو وتكون زاوية دنل مساوية لزاوية دطز ولأن خط دس أطول من خط ند تكون زاوية دنس أعظم من زاوية دسن فزاوية دسم الباقية أعظم من زاوية دنل الباقية التى هى مثل زاوية دطز فإذا أخرجنا من نقطة ط خطاً موازياً لخط مس سيقع طرفه الآخر تحت نقطة ز وهو خط طك فنسبة سد إلى طد كنسبة مس إلى طك فهو أطول طك وطك أطول من طز فمس أطول من طز الذى هو مثل نل، وأيضاً فإنا نبين ذلك دون أن نخرج العمود بأن نقول إن زاوية دزط التى هى مثل زاوية دلن

and through the degree of the star in the zodiac and bisecting the zodiac, then it will also bisect the circle of the equator. Where this arc intersects with the circle parallel to the zodiac, this point is the position of the star on the plate. In this manner are constructed the circles parallel to the horizon circle, which are the almucantars — the demonstration of them [*sc.* the almucantars] rests on them [*sc.* the zodiac-parallels].

[8] Maslama said about the proposition that begins "We must prove that the centre of the circles parallel to the circle of the signs which are drawn in this manner are always different" and ends "and point O is not the centre of the circle passing through points N, C. Q.E.D.": [Ptolemy] did not prove that the centres were not the same point except after proving that line MC is longer than line NL; and we shall show that in an easier way than this. That is that we repeat the figure, and that is that from point N we draw a line that is perpendicular [72v] to line DL, which is NY, and prove that DN is shorter than DC and DL is shorter than DM. Let us cut off from line DC a line, DT, equal to DN and from line DM a line, DZ, equal to DL. Then triangle DTZ is equal to triangle DNL, perpendicular NY falls like perpendicular TO, and angle DNL is equal to angle DTZ. Because line DC is longer than line ND, angle DNC is greater than angle DCN. Therefore angle DCM, remaining, is greater than angle DNL, remaining, which is equal to angle DTZ. When from point T we draw a line parallel to line MC, its other end will fall beneath point Z — it is line TK. Therefore the ratio of CD to TD is as the ratio of MC to TK. Therefore it is longer than TK. But TK is longer than TZ: therefore MC is longer than TZ, which is equal to NL.

We can also prove that without drawing the perpendicular by saying that angle DZT, which is equal to angle DLN,

حادة فخط كط أطول من خط زط، ومن هذا الشكل يتبين أن الأقسام التى تقسمها الخطوط المستقيمة المخرجة من نقطة د فى خط جا على قسى متساوية تكون فى خط جا غير متساوية وأن كل ما قرب من المركز كان أقصر مما بعد عنه .

(Figure 3)

<٩> <73r> قال مسلمة فى الشكل الذى أوله »ويجب الآن لمكان الدائرة الموازية لدائرة فلك البروج التى ليست محصورة فى الصفيحة لكن يقع بعضها فى القطعة التى لا تظهر وهى غير مرسومة فى الكرة« وفى آخره »وقسمت دائرة نعق على نقطة ع بقوسين مشابهتين لقوسى حم مز«، ومن تمام هذه المسئلة هق على استقامة ودط على استقامة حتى يلتقيا على نقطة و ويبرهن أن الدائرة التى تمر بنقطة س ونقطة ع تمر بنقطة و أيضاً على ما برهن فى المسئلة التى قبل هذه فى وضع الدائرة الموازية لدائرة فلك البروج التى لا يقع فى الدائرة الخفية منها شىء فتتم له فى هذه المسئلة جميع الشروط التى تمت له فى التى قبلها، فإذا أردنا ذلك فإنا نصل دز ونخرجه على استقامة حتى ينتهى إلى نقطة ق على ما انتهى دح إلى نقطة ن ونصل دك ونخرجه على استقامة إلى ر وكذلك أيضاً نخرج خط زح على استقامة إلى نقطة ت فخط ون قد انقسم بمثل أقسام خط تح فى النسبة لموازاة حت لون ونجيز على نقطة ط خطاً موازياً لزح وهو طص فقوس دص مساوية لقوس دط فزاوية دطص مساوية لزاوية دلط وزاوية دطص هى مساوية لزاوية دتح للموازاة فزاوية دلط مساوية

is acute, and so line KT is longer than line ZT.

From this proposition it is clear that the divisions that the lines drawn from point D divide off in line GA [and standing] on equal arcs are unequal in line GA and that the nearer they are to the centre, the shorter they are [contrary to] when they are farther from it.

[Fig. 3]

[9] [73r] Maslama said about the proposition that begins "Now it is necessary for the place of the circle parallel to the zodiac circle that is not contained in the plate, but falls partly in the section that is not visible, which is not drawn in the sphere" and ends "Circle NYQ is divided at point Y into arcs similar to arcs HM, MZ": to complete this question EQ [is produced] in a straight line and DT [is produced] in a straight line until they meet at point O. He proves that the circle that passes through point C and point Y also passes through point O, as he proved in the question that precedes this one about the position of the circle parallel to the zodiac circle of which nothing falls inside the hidden circle. In this question all the conditions are fulfilled that were fulfilled in the one preceding it. When we want that, we join DZ and produce it in a straight line until it reaches point Q, just as DH reached point N, and we join DK and produce it in a straight line to R; similarly, we also produce line ZH in a straight line to point P. Line ON is partitioned into the same divisions, proportionately, as line PH, because of the parallelism of HP to ON. Through point T we draw a line, TX, parallel to ZH. Therefore arc DX is equal to arc DT; and therefore angle DTX is equal to angle DLT. Therefore angle DTX is equal to angle DPH because of parallelism. Therefore angle DLT is equal

لزاوية دتح فنقط ل ث ط ت على محيط دائرة فضرب تك فى كث كضرب <73v> كط فى كل وقد كان ضرب طك فى كل مساوياً لـكز فى كح فـكز فى كح مساوٍ لـكت فى كث فضرب إذن قر فى رن كضرب رو فى رس ونصل رع فمثلث رهع شبيه بمثلث كفم لأن زاوية ق مثل زاوية ه والخطان المحيطان بها متناسبة لأن نسبة ره إلى فك كنسبة هق إلى فز للموازاة فزاوية عره إذاً قائمة فضرب قر فى رن مثل رع فى مثله وقد كان قر فى رن كضرب رو فى رس فضرب رو فى رس مثل ضرب رع فى مثله ورع عمود فنقط سعو على محيط دائرة، وذلك ما أردنا أن نبين .

(Figure 4)

<١٠> قال مسلمة أيضاً ولكى نكمل ما يجب أن نكمله فى هذه المسئلة وذلك أن نبين أن هذه الدائرة الموازية لدائرة فلك البروج ستقطع أيضاً بنصفين الدائرة الموازية لدائرة معدل <74r> النهار التى تقطعها فى الكرة المجسمة بنصفين، برهان ذلك أنا نجعل دائرة نصف النهار دائرة ابجد حول مركز ه والقطب الخفى نقطة د وقطر الدائرة الموازية لدائرة فلك البروج طكل ونصل دل ونخرجه إلى س ودط ونخرجه إلى و ثم نجيز على نقطة ك خطاً موازياً لقطر دائرة معدل النهار الذى هو اج وهو خط ن ح كز ونصل دح ونخرجه على استقامة إلى ق فمما قد تقدم أن الدائرة التى ترسم ببعد هق هى الدائرة الموازية لدائرة معدل النهار التى بعدها منها قوس اح وهى الدائرة التى تقسمها الدائرة الموازية لدائرة فلك البروج فى الكرة بنصفين على خط نصف النهار ثم نخرج هب إلى ف وليكن هف مثل هق فبيّن أن الدائرة التى تمر بنقط سوع ستمر بنقطة ف

to angle DPH. Therefore points L, S, T, P are on the circumference of a circle. Therefore $PK \cdot KS =$ [73v] $KT \cdot KL$. But $TK \cdot KL = KZ \cdot KH$: therefore $KZ \cdot KH = KP \cdot KS$. $QR \cdot RN$ therefore $= RO \cdot RC$. We join RY: triangle REY is similar to triangle KFM, because angle F is equal to angle E and the lines including them are proportional, because the ratio of RE to FK is as the ratio of EQ to FZ because of parallelism. Angle YRE is therefore right, and so $QR \cdot RN = RY^2$. But $QR \cdot RN = RO \cdot RC$. Therefore $RO \cdot RC = RY^2$. And RY is perpendicular [to OC]: therefore points C, Y, O are on the circumference of a circle. Q.E.D.

[Fig. 4]

[10] Maslama also said: in order to complete what we must complete in this question, and that is that we show that this circle parallel to the zodiac circle will also bisect the circle parallel to the equator circle [74r] which it bisects in the solid sphere. *Proof*: we make the meridian circle circle $ABGD$ about centre E, the hidden pole point D, and the diameter of the circle parallel to the zodiac TKL; we join DL and produce it to C and [join] DT and produce it to O. Then through point K we draw a line, $NHKZ$, parallel to the diameter of the equator circle, AG. We join DH and produce it in a straight line to Q. From the above [it is clear] that the circle that is drawn with distance EQ is the circle parallel to the equator circle whose distance from it is arc AH — it is the circle that the circle parallel to the zodiac circle bisects in the sphere on the meridian line. Then we produce EB to F; and let EF be equal to EQ. It is clear that the circle that passes through points C, O, Y will pass through point F.

فنجيز على نقطة ط خطاً موازياً لخط كح وهو طم فزاوية دطم مساوية لزاوية دلط لأنهما على قوسين متساويتين وزاوية دطم مساوية لزاوية دنك للموازاة فزاوية دنك مساوية لزاوية دلط فنقط ل ز ط ن على محيط دائرة فضرب كن فى كز مساوٍ لـكط فى كل وكط فى كل مساوٍ لـكح فى مثله فـكن فى كز مثل كح فى مثله وقد انقسم خط وس بمثل انقسام خط نز فى النسبة فيجب أن يكون ضرب وه فى هس كضرب قه فى مثله وقه مثل هف فضرب وه فى هس كضرب <74v> هف فى مثله فالدائرة التى تمر بنقط س ع و تمر أيضاً بنقطة ف، وذلك ما أردنا بيانه، وعلى مثل هذه الرتبة نخرج الدائرة الموازية لدائرة الأفق الواقعة تحتها لا فرق بينهما فتكون المقنطرات الواقعة فى الصفيحة على الأسطرلاب الجنوبى خط دلس مستقيم وخط طل مستقيم .

(Figure 5)

<١١> قال مسلمة فى الشكل الذى أوله »ومن البيّن أنا إن توهمنا فى مثل هذه الصورة الدائرة الموازية لدائرة فلك البروج التى على قطرها دل« وفى آخره »وذلك أن سطح دائرة نصف النهار أيضاً التى على خط بد هو على زوايا قائمة على كل واحد من السطحين اللذين ذكرناهما«، <75r> وأريد أن هذا الخط المستقيم يقطع الدائرة الخفية بقسى شبيهة بالقسى التى تقسمها فى الكرة المجسمة فى شكل <...> وذلك أن نعيد الصورة وليكن قطر الدائرة الموازية لمعدل النهار الخفية خط زكح فالدائرة المرسومة ببعد هن خفية أبداً ونقيم على خط زح نصف دائرة ونخرج من نقطة ك خطاً موازياً لـهد وهو كم

Through point T we draw a line, TM, parallel to line KH. Angle DTM is equal to angle DLT, because they are on equal arcs, and angle DTM is equal to angle DNK because of parallelism. Therefore angle DNK is equal to angle DLT. Therefore points L, Z, T, N are on the circumference of a circle. Therefore $KN \cdot KZ = KT \cdot KL$. But $KT \cdot KL = KH^2$: therefore $KN \cdot KZ = KH^2$. But line OC is partitioned like line NZ, proportionately. Therefore $OE \cdot EC$ must be equal to QE^2. But $QE = EF$: therefore $OE \cdot EC =$ [74v] EF^2. Therefore the circle that passes through points C, Y, O also passes through F. Q.E.D.

In the same way we draw the circle parallel to the horizon circle and lying under it. There is no difference between them. They are the almucantars occurring in the plate on the southern astrolabe. Line DLC is straight; line TL is straight.

[Fig. 5]

[11] Maslama said about the proposition that begins "It is clear that if we imagine, in a diagram similar to this one, the circle parallel to the zodiac circle on whose diameter is D, L" and ends "and that is because also the plane of the meridian circle, which is on line BD, is at right angles to each of the two planes that we mentioned": [75r] it is wanted that this straight line cuts the hidden circle in arcs similar to the arcs that it cuts off in the solid sphere in proposition [. . .]. That is that we repeat the figure. Let the diameter of the hidden circle parallel to the equator be line ZKH; and so the circle drawn with distance EN is always hidden. On line ZH we erect a semicircle; and from point K we draw a line, KM, parallel to ED.

فالدائرة الموازية لدائرة فلك البروج المرسومة على قطر دل تقطع فى الكرة الدائرة الخفية على نقطة م فتقسمها بقوسين وهما حم ومز وكذلك يقسم خط بع دائرة نعق بقوسين شبيهتين بقوسى حم مز وهما نع عق، برهان ذلك أنا نصل هع قم فمن أجل أن خط زح موازٍ لنه تكون نسبة نه إلى سه كنسبة حف إلى فك ونه مثل هع وحف مثل قم فنسبة عه إلى هس كنسبة مف إلى فك وزاوية عسه قائمة وهى مساوية لزاوية مكف يكون مثلث مفك شبيهاً بمثلث عهس وتكون زاوية عهس مساوية لزاوية مفك فقوس نع شبيهة بقوس حم وكذلك الباقية من نصف الدائرة وهى قوس عق تكون شبيهة بالباقية من نصف الدائرة وهى قوس مز فقد قسم خط بع دائرة <75v> نعق بقوسين شبيهتين بالقوسين اللتين تقسم الدائرة الموازية لفلك البروج <من> الدائرة الخفية فى الكرة المجسمة، وأيضاً فإن هذه الدائرة تمر فى الكرة بالقطب الخفى ولن يقع فى هذا السطح القطب الخفى مع القطب المرئى إذ سطح هذا القطب يمتد إلى ما لا نهاية ولا يبلغ إلى القطب الجنوبى وكذلك خط بع إن أخرج إلى ما لا نهاية لم يلتق مع خط جد، وذلك ما أردنا بيانه .

(Figure 6)

تم تعاليق مسلمة بن أحمد على كتاب بطلميوس فى تسطيح بسيط الكرة والحمد لله وحده وفرغ من تعليقه بظاهر أسدآباد بالمعسار المنصور العابى فى د يا ح ثمد وصلى الله على سيدنا محمد .

Therefore the circle parallel to the zodiac circle drawn on diameter DL cuts in the sphere the hidden circle at point M, dividing it into two arcs, HM and MZ. Similarly, line BY divides circle NYQ into arcs similar to arcs HM, MZ — they are NY, YQ. *Proof*: we join EY, FM. Because line ZH is parallel to NE, the ratio of NE to CE is as the ratio of HF to FK. But NE is equal to EY and HF is equal to FM: therefore the ratio of YE to EC is as the ratio of MF to FK. Angle YCE is right and is equal to angle MKF: [therefore] triangle MFK is similar to triangle YEC. [Therefore] angle YEC is equal to angle MFK. Therefore arc NY is similar to arc HM. Similarly, the remainder of the semicircle, arc YQ, is similar to the remainder of the semicircle, arc MZ. Line BY has divided circle [75v] NYQ into two arcs similar to the arcs that the circle parallel to the zodiac cuts from the hidden circle in the solid sphere.

Now this circle also passes, in the sphere, through the hidden pole. The hidden pole is not found in this plane with the visible pole, since the plane of this pole [may be] extended *ad infinitum*, but it does not reach the south pole; and accordingly, if line BY is produced *ad infinitum*, it does not meet line GD. Q.E.D.

[Fig. 6]

The comments of Maslama ibn Aḥmad on Ptolemy's Book on the Projection of the Surface of the Sphere are finished. Praise to God alone! Al-Manṣūr al-ʿ.ā.ī[4] completed its writing in the suburbs of Asadābādh in the miʿsār [?] on Wednesday, the 11th of the 8th [month] 544 [A.H. = 14th December 1149]. May God bless our lord Muḥammad.

[4] Al-ʿAttābī, or al-ʿAnānī ?

Apparatus
MS = Paris, Bibliothèque Nationale, ar. 4821 (Maslama)
Y = Istanbul, Aya Sofya 2671 (*Planisphaerium*)
T = Tehran, Khān Malik Sāsānī (T is only mentioned when different from Y)
The apparatus follows the line numbers of the above edition.

(4-7) «أعنى ... دائرة فلك البروج» = Y 78r,10-14; 6 أوضحنا: قد أوضحناه Y

(17-18) «أنا ... نقطة ط» = Y 79v,9-10; 17 خط هح: خط جه Y; 18 استقامة:
الاستقامة Y; دائرة الأفق: دائرة حاج وهى دائرة الأفق Y

(28) دط: هز MS

(30) وثمنون: وثمنين MS

(42-44) «فإذ ... المجسمة» = Y 83r,15-17; 42 فإذ: وإذ Y; 43 وصفنا:
وضعنا Y; فنبين: فلنبين أن Y; أنه: أيضاً Y; 44 على ما: على مثال ما Y

(44-45) «فزاوية ... معلومة» = Y 86r,1; 45 وذلك ... نبين: *deest in* Y

(51-53) «ولكن ... ذلك» = Y 92r,4-6; 52 ذكرنا: *sic* T, ذكرناه Y; 53 لنا: لنا
فيه T; مواضع: وضع Y

(53-54) «شبيهة ... جز» = Y 92v,16; 54 جز: بجز فقوس من الباقية شبيهة
بقوس دز الباقية Y; 54 وذلك ... نبين: *deest in* Y

(66) حكط: حطط MS

(68-70) «وأيضاً ... النهار» = Y 92v,ult.-93r,2; 68 ينبغى وأيضاً: وينبغى
أيضاً Y; 69 بوسط: فى وسط Y

(70-71) «فإنها ... أيضاً» = Y 93v,1-2; 71 الأخرى: الأجزاء MS

(77-78) «وقد يمكننا ... المثال» = Y 93v,2-5; 78 فلك البروج: البروج Y

(78-79) «فنقط ... دائرة» = Y 94r,10-11; 78-79 فنقط ن ع م ف: فنقطة م ع ن ف
إذاً أيضاً Y; 79 محيط دائرة: محيط بدائرة واحد خط Y

(86-87) «وينبغى ...أبداً» = Y 94r,11-13; 87 البروج: البروج أيضاً Y

(88-89) «فليس ... نبين» = Y 95r,5-7; 88 نقطة و: نقطة ف Y; 88-89 لمركز
... نبين: مركز الدائرة التى يكون خط يس [T سن] قطراً لها Y

(91) دل: دم MS

(94) لمثلث دنل: لمثلث دنم MS

(105-107) «الكرة ... ويجب» = Y 95r,8-10; 105-106 الدائرة الموازية :الدوائر
في الموازية Y; 106 البروج :فلك البروج Y; ليس هي :ليست Y; 107
الكرة :من الكرة Y

(107-108) «مز حم ... وقسمت»: *discedit a textu* Y; 108 مشابهتين:
مشابهين MS; حم مز :جم مر MS

(120) مساوياً :مساوٍ MS

(122) زاوية ب :زاوية ق MS

(123) فك :بك MS; فز :بز MS

(148-149) «دل ... ومن البين» = Y 95v,9-11; 148 إن توهمنا :وإن توهمنا Y;
149 البروج :فلك البروج Y; دل ... التي :نقطة د التي ترسم على Y

(150-151) «ذكرناهما ... وذلك» = Y 95v,ult.-96r,3; 150 خط بد :خط اج Y,
خط اح T; هو :هي Y; 150-151 على زوايا قائمة: *sic* T, زوافاىمة Y; 151
السطحين :هذين السطحين Y

2. Maslama's notes, Latin versions

The manuscripts

Class I (notes within text: i.e. 1, 2, 3, 6, 7, 9 [except the beginning], 11):

D Paris, Bibliothèque Nationale, lat. 7399, 14c., ff. 1r–12v.

C Paris, Bibliothèque Nationale, lat. 7214, 14c.?, ff. 211r–217v.
K Krakow, Jagellonian Library, 1924, 13c., pp. 165–189.
L Lyon, Bibliothèque Nationale, 328, ff. 47r–58r.
W Vienna, Nationalbibliothek, 5496, 14–15c., ff. 1r–11v.

M Milan, Ambros. A 183 inf., ff. 14r–19v.
S Escorial, d. II 5, ff. 64r–71v, 16c.?
X Printed edition, Venice 1558, ed. Commandinus (copy used: Oxford, Bodleian Library, Savile Aa3).

The manuscripts have been grouped according to their treatment of the *Propositiones planisperii*. For more details see below, Appendix II. **W** is very close to **K**. In fact it is probably copied from it, for on p. 181 the diagram obtrudes into the text, the diagram letter O coming directly over KC of the text, and **W** has KOC in this place (see text, note 9^3, line 18 and apparatus).

Class II (notes as in Class I plus 5, 8, 9 [beginning only] and 10 in the second translation: in **AB** 5, 8, 10 at end, 9 *marg.*; in **NP** 5 and 10 in text [10 inserted in the middle of note 9 of Class I], 8 in text in **P**, but missing in **N** [part of text is also missing], 9 *marg.* in **N**, but missing in **P**; Extra-Chapter, in the same, second, translation, after the notes):

A Vatican, Reg. lat. 1285, early 13c, ff. 153r–162r.
B Vatican, lat. 3096, ff. 3r–14r, 14c.

N Paris, Bibliothèque Nationale, lat. 7377B, 13–14c., ff. 73r–77, 80, 78, 61–62, 81r–v.
P Parma, Palatine Library, 954, 15c., ff. 106r–155r.

B is close to **A**; **P** is usually close to **N**.

Class III (notes as in Class I plus 4, 5, 7 [part only], 8, 9 [beginning only], 10, all *marg.*, in the third translation; Extra-Chapter and Astrolabe Chapters, in the same, third, translation, separate from the *Planisphaerium* and notes; the margins also contain alternative translations of short passages from Maslama's notes and the *Planisphaerium* itself):

E Oxford, Bodleian Library, 13c., Auct. F.5.28, ff. 88r–95r, 55v–57v.
F Dresden, Sächsische Landesbibliothek, Db 86, ff. 214r–219v, 14c. This manuscript, which is close to **E** in the parts that can be read, is seldom used in this edition because of water damage; only positive indications in the apparatus are to be taken as readings from **F**.

The sigla **A**, **B**, **C**, **D**, **E**, **F** are the same as in Heiberg's edition of the *Planisphaerium*.

The orthography of **A** has been followed. Orthographic variants have not been noted.

There are diagrams in the Latin manuscripts as well as in the Arabic text. For the edition we have reconstructed them from **Ar.**, **A** and **E** and placed them in the Latin section, in the appropriate places, with diagram letters according to **ABNP**, whose system of transliteration is the same as Hermann's in his translation of the *Planisphaerium*. For further details see the first section of the Comments.

Ar. indicates the Arabic text edited above.

< 1 >

A 154rb–va, **B** 3v, **N** 78v, **K** p. 168, **C** 211vb, **D** 2v–3r, **X** 3r, **E** 89r, **M** 15r, **S** 64r marg., **W** 2v, **L** 48v, **P** 107r–v

In hunc locum maslem commentans ait ut descriptis equidistantibus recto hinc inde circulis deducatur zodiacus: et ubi singulos interceperit, signorum initia statuantur. Quo artificio singulorum etiam graduum initia constitui possunt.

< 2 >

A 155ra, **B** 4r–v, **N** 79r, **K** p. 169–170, **C** 212ra–b, **D** 3v, **X** 5r, **E** 89v, **M** 15r, **S** 64v, **W** 3v, **L** 49r, **P** 107v–108r

Addit maslem argumentum: lineam HE in directum ductam non posse orizontem preter punctum T attingere. Esto enim ut alterutra ex parte attingat atque, si placet, ad punctum M producaturque in directum EM usque ad circumferentiam zodiaci in punctum Z. Quoniam igitur quanta est AE in EG tanta HE in EM, erit et quanta BE in ED; est autem quanta BE in ED tanta HE in EZ. Eiusdem est igitur HE in EM et HE in EZ; unde EM et EZ equales esse consequens est. Inpossibile est ergo lineam HE in directum productam orizontem preter punctum T attingere. Ex his consequens est quod omnis circulus, qui alterutrum horum per medium secat, et alterum per equalia secabit.

---

1 maslem] maxime **CP**, maslen **DE**, i *add. et del.* **E** *1* ait] sit *add.* **M** *1* descriptis] descriptus **B** *2* inde] *om.* **P** *3* interceperit] intercepit **M**, inceperit **P** *3* statuantur] sumantur **KW** *3* artificio] et *add.* **X** *4* etiam] et **CMSW**, in **P** *4* graduum initia] initia graduum **DE** *4* constitui possunt] possunt constitui **C** 2: *1* argumentum lineam] *om.* **C**; lineam: linea **B** *1* *HE*] *om.* **M**, *AE* **P** *1* directum] *corr. ex* directa ? **N** *1* ductam] ducta **B** *2* posse] post se **W** *2* Addit ... orizontem] *marg.* **C** *2* *T*] *om.* **M** *2* enim] *om.* **B** *3* alterutra ex parte] ex parte alterutra **KW**, ex parte **DES**, ex parte altera **XM**, alterutra **P** *4* in] quia **M** *4* *EM*] *EN* **NP** *4* ad] in **KXW** *5* igitur] ergo **BX**, itaque **C**, ? **S** *5* in] et **NP** *6* *HE*] *AE* **L** *6* *BE*] *NE* **P** *6* est] *om.* **B** *7* *BE*] *EB* **W** *7* *BE* in *ED* tanta] *EB* in *ED* tanta *marg.* **K**, *om.* **DXEMS** *7* igitur] ergo **X** *8* EZ^1] Eiusdem est ... *EZ* *om.* **L** *8* unde] si **S** *8* *EM*] *EG* **M** *8* consequens est] est consequens **C**, consequitur **L** *9* et *EZ* ... productam] *marg.* **E**, *om.* **S** *10* attingere] attinget **S** *10* consequens est] est consequens **C** *11* omnis circulus] omnem circulum **L** *11* medium] *corr. ex* (?) **S** *12* et alterum] alterutrum **C** *12* equalia] *corr. ex* media **A**, medium **L** *12* secabit] secare **L**

< 3 >

A 155vb–156ra, B 5v, N 76r, K p. 171–172, C 212vb–213ra, D 4v, X 8r–v, E 90v, M 16r, S 65v, W 4v, L 50v, P 198v

Hic locus est argumenti maslem: quia deprehensum est, inquid, quota distantia equidistantes recto circulo terminant linee DTZ et DKH, ut semidiametros australis circuli a puncto E porrigatur usque quo linea DT concurrit cum linea EG. Velud si arcum GT ponamus etiam gradus lxxxviiii, necesse est linearum concursum fieri super diametrum circuli distantis ab equinoctiali ad austrum gradibus lxxxix. Scimus autem distantiam poli ab equinoctiali circulo integris xc gradibus, quantus est totus arcus GD. Si ergo in hac planicie polum australem invenire debeamus, illic oportet qua lineam EG equidistans ei a puncto D producta contigerit. Equidistantes vero nunquam concurrunt: ergo inpossibile est in hac planitie australem representari polum. Nam nec si polum australem posuerimus, septentrionalem adesse possibile est. Si enim linee recte per positum polum transeuntes eos notant circulos qui

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1 argumenti] argumenta **ABNP**, argenti **C**; est argumenti: argumenti est **E** *1* maslem] maslen **DE** *1* Hic locus ... est] Hunc locum vinctitatur in quo dixit deprehensum esse **S** *2* inquid] inquam **L** *2* quota distantia] quotam distantiam **S** *2* equidistantes] equidistans **C**, equidistantis **L** *2* circulo] *om.* **KW**; recto circulo: circulo recto **L** *3* DTZ] DCZ **C** *3* australis circuli] circuli australis **L**; paralleli *supra* **E** *4* quo] qua **ACM**, quam **BNP** *4* DT] DC **C**, TD **X** *4* concurrit] concurrat **KXWL** *4* concurrit cum linea EG] cum linea EG concurrit **D**, cum linea EG concurrat **E**, cum linea EG concurrerit **S** *5* velud si] vel desit **C** *5* si arcum] suarum **P** *5* etiam] *om.* **XL**, et **M** *5* lxxxviiii] 89 *corr. ex* 88 **B**, 69 ? **M** *6* concursum] concresum **P** *7* distantis] equidistantis **SL** *7* ab] *om.* **C** *7* austrum] austri **P** *7* lxxxix] 99 ? **M** *7* scimus] sumus **B** *8* circulo] *om.* **L** *8* integris] integer **P** *9* est] *om.* **XM** *9* totus] *om.* **KW** *9* arcus GD] DG arcus **KW**, GD arcus **X** *9* hac] linea **M** *9* planicie] planiciei **S**; nitie *add. et del.* **A** *10* polum australem] australem polum **S** *10* debeamus] debemus **DE**, deberemus **S** *10* oportet] oporteret **S** *10* qua] quam **B**, ubi **XS** *10* lineam] linea **P** *11* equidistans] equidistat **P** *11* ei] *corr. ex* EC **C** *11* producta] puncta **S** *11* contigerit] contingeret **NP**, contingit **KDXESW**, continget **M** *12* ergo] igitur **DE** *12* hac planitie] hanc planitiem **XM**, plano **L** *13* australem representari polum] polum australem representari **KXW**, australem polum representari **ML** *13* polum[2]] *om.* **L** *14* posuerimus] posuimus **CP** *14* septentrionalem adesse] adesse septentrionalem **X** *14* possibile] impossibile **P** *14* est] *supra* **K**, *om.* **DE**, esset **S** *14* Si] Sic **M**; Amplius *supra* **E** *15* linee recte] recte linee **M** *15* positum] *om.* **DESP**; per positum: propositum **XM** *15* polum] *supra* **E**

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14 Nam nec si ... est] Et iam secundum hoc si posuerimus notam E polum meridianum, erit inpossibile septentrionalem simul in eadem consistere superficie *marg.* **E** (*cf.* **Ar.**)

sese ad utrumque polum intersecant, si uterque adesset, eas lineas duobus in locis sese intercipere necesse foret. Quod quoniam in rectis lineis inpossibile est, nec in una planitie utrumque representari polum possibile est.

< 4 >

E 91v marg., F 216v marg.

Dixit meslem: et est ad hoc etiam via facilior, quia ipse memoravit diametros parallelorum equalis ab equatore longitudinis in partem septentrionis et meridiei. Et quod erit linea *EK* semidiameter paralleli meridiani, et linea *EN* septentrionalis. Et iam assignavit quantitates diametrorum eorum in precedentibus et demonstrationem in propositione secunda libri huius.

< 5^1 >

A 160va–b, B 11v–12r, N 78v, P 111r–v

Dixit maslem: et si intenderet ptolemeus in hac questione ad id quod rememorabor, iam foret acceptio proprior. Quod est quando continuatur *D* cum *Z*, si protraheretur a puncto *G* linea equidistans linee *DZ*, que est linea *GM*, deinde fieret circulus, qui est revolutio capitis arietis, secundum longitudinem *EM*, esset circulus arietis. Et illud est quoniam angulus *NME* equalis est angulo *ZDE*, arcus igitur *BGZ* est similis arcui *CLN*; remanet ergo arcus *NL* similis arcui *GZ*. Et illud est quod voluimus demonstrare.

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16 transeuntes eos ... polum] *om.* **P** *16* si uterque] necesse foret si uterque polorum **S** *17* duobus] rectas *add.* **S** *17* duobus in] in duobus **XML**, duobus **P** *17* sese] se **L** *17* necesse foret] *om.* **S** *17* Quod] *supra* **N** *18* quoniam] possibile est *add.* **B** *18* in] *om.* **KSW** *18* nec] non **DE**; inpossibile est nec: non erit possibile **S** *18* una planitie] uno plano **L** *19* planitie utrumque representari] representari planitie utrumque **KXMW** *19* possibile est] *om.* **BS** 4: *1* meslem] maslem **F** *1* ad hoc etiam] etiam ad hoc **F** *5* iam] ideo **F** 5^1: *1* maslem] mallem **P** *2* Quod] que **B** *4* *G* linea] *GB* a **P** *4* *DZ*] *ZD* **P** *6* *EM*] *A* **P** *6* illud] id **P** *7* *ZDE*] *ZD* **P** *7* *BGZ*] *EGZ* **P** *9* voluimus demonstrare] v. de. **AP**, d. voluimus **B**

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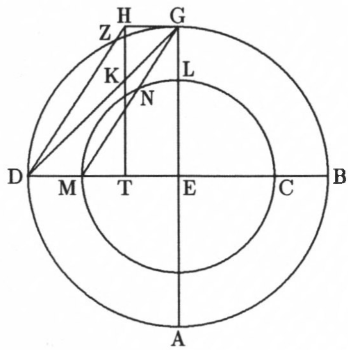
17 duobus in locis] uno videlicet polo septentrionali et altero meridiano *marg.* **E** (*cf.* Ar.)

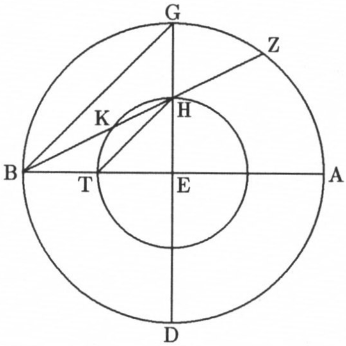


[Fig. 1]

Et est ei alius modus propinquior. Quod est ut ponamus maiorem ex circulis cadentibus in instrumento circulum $ABGD$ circa centrum E et protraham duas diametros circuli sese secantes ortogonaliter. Et volumus describere infra ipsum circulum cuius circuli longitudo a circulo descripto infra ipsum sit secundum longitudinem arcus similis arcui GZ. Deinde continuabimus B cum Z, et secet lineam EG super punctum H. Et ponemus E centrum et describemus circulum <secundum> longitudinem EH, qui est circulus HKT. Dico igitur quod arcus KT est similis arcui ZG. Et probatio eius est quia continuabimus G cum B et H cum T: HT ergo est equidistans GB; angulus igitur KHT est equalis angulo GBZ. Et uterque eorum est super circumferentiam circuli: arcus igitur KT est similis arcui GZ. Cum igitur fuerit circulus HKT equator diei, erit circulus $ABGD$ descriptus super ipsum secundum longitudinem arcus KT, qui est similis arcui GZ. Et illud est quod demonstrare voluimus.

---

10 ei] *supra* **B** *10* modus] *corr. ex* circulus **B** *12* centrum] est *add. et del.* **A** *12 E*] *supra* **B** *12* protraham] producam **B** *12* duas] duos **BP** *13* describere] *marg.* **B** *15* arcui] *seq. ras.* **P** *16 B*] *G* **A** *16* lineam] linea **NP** *17 E*] est **N** *17* et] *om.* **P** *19 ZG*] *GZ* **B** *20 G* cum *B*] *B* cum *G* **BP** *20 H*] *corr. ex HT* **A** *20* ergo] e *add. et del.* **B** *20* est] *E* **P** *21* Et uterque] ut alter **P** *22 KT*] *BT* **B**, *HT* **P** *23* arcui] arcus **MSS** *26* quod demonstrare voluimus] qd'. d'. v. c. **A**, q.d.v. **B**; et illud ... voluimus *om.* **P**



[Fig. 2]

< 5^2 >

E 93r marg., **F** 218r marg.

Dixit meslem: si animadvertisset Ptholomeus in hac propositione, propinquior esset assumptio, quod deus velit, et hoc est quando adiunxit *D* cum *Z*, si duxisset a nota *G* lineam equidistantem linee *DZ*, que est *GM*, et fecisset circulum cum distantia *EM*, esset circulus arietis. Quia angulus *NME* equalis angulo *ZDE*, ergo arcus *BGZ* similatur arcui *CLN*.

Et ad illud aliter: quia ponam maiorem circulorum *ABGD* circa centrum *E* et ducam diametros orthogonaliter se secantes. Volumus ergo signare intra ipsum cum longitudine arcus similis arcui *GZ*. Et continuabimus *B* cum *Z*, et secabitur *EG* super *H*. Fiatque circulus secundum *EH*, centro super *E*, et sit *HKT*. Arcus ergo *KT* similis est arcui *ZG*, quod ductis equidistantibus *GB* et *HT* manifestum est.

< 6 >

A 158vb, **B** 9r, **N** 78v, **K** p. 178, **C** 215rb–va, **D** 8r, **X** 19r–v, **E** 93r–v, **M** not in the available film, **S** 68v, **W** 7v–8r, **L** 54r, **P** 111v

Hic subiungit maslem quod cum huiusmodi circulus in planisperio describatur, si per gradum stelle transeat ubicumque

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1 meslem] maslem **F** *6* equalis] equatur **F** 6: *1* maslem] maslen **DE** *2* planisperio] planis sperio **N** *2* ubicumque] utcunque **X**

---

9 ipsum] *hic addendum* circulum (*cf.* **Ar.**)

sita sit, transire quoque necesse habet per ipsum corpus stelle. Converso quoque si per corpus stelle transeat, transibit etiam per gradum stelle. Amplius linee recte per centrum equinoctialis circuli in planisperio transeuntes si per corpus stelle transeant, transibunt et per gradum cum quo celum mediat, id est in quo ipsa meridianam transit lineam. Converso quoque si per hunc gradum transeant, transibunt etiam per ipsum corpus stelle ubicumque sita fuerit.

< 7^1 >

A 159ra, **B** 9v, not in **N**, **K** p. 179, **C** 215va–b, **D** 8v, **X** 20v, **E** 93v, **M** not in the available film, **S** 69r, **W** 8r, **L** 54v, **P** 112r

Noto igitur, ut maslem addit, circulo equidistante zodiaco distantia eius latitudinem stelle metitur. Quo firmato deducemus a polo *K* zodiaci in supradata descriptione notato arcum per gradum stelle in zodiaco tam zodiacum quam equinoctialem per medium secantis circuli. Ubi ergo hic arcus equidistantem zodiaco secuerit, is punctus est stelle locus in planisperio. Hac constitutione de equidistantibus zodiaco habita simili ratione isdemque argumentis constitui possunt et equi-

---

3 ubicumque sita] uterque **P** *3* sit] erit **S** *3* necesse habet] habeat **KW**, hanc **C**, hunc **X**, necesse habeat **S** *3* ipsum] ipsam **C** *4* Converso] *corr. ex* Conversio **B** *4* Converso quoque] et **KDXESWL** *4* per] ipsum *add.* **KXW** *4* etiam] et **DE**, quoque **L** *5* Converso ... stelle] *om.* **C**; transibit etiam ... stelle: et gradum stelle transibit **S** *6* circuli] *om.* **L** *6* planisperio] planispera **N**, planisspera **P** *6* transeuntes] transiens **BS** *7* et] etiam **BC** *7* mediat] mediant **L** *8* id est] *om.* **B** *8* in] cum **KXESWLP** *8* quo] *om.* **P** *8* ipsa] ipsam **BC** *8* transit] transibit **KDESWP**; meridianam transit: transibit meridianam **X** *8* Converso quoque] eodem modo **L**; modo (?) *add.* **S** *9* transeant] *hic desinit* **N** *9* ipsum] *om.* **C** *10* stelle] *om.* **C** *10* fuerit] sit **L** 7^1: *1* igitur] ergo **ABK** *1* Noto igitur ut] *om.* **X** *1* maslem] mallem **CP**, maslen **E**, *corr. ex* mallea **S** *1* equidistante] equidistantem **KDEW**, equidistantes **P** *1* zodiaco] zodiaci **P** *2* distantia] equidistantia *corr. ex* equidem stantia **S** *2* eius] eis **P**; distantia eius: cuius distantia **X** *2* Quo] *om.* **X** *3* deducemus] ducemus **BP** *3* *K*] *om.* **KCDXESWL** *3* zodiaci] *om.* **EL** *3* supradata] supradicta **CDESP** *3* notato] nota **P** *5* circuli] Verbi gratia *add. et del.* **W** *5* ergo] igitur **KCDEWL**; ubi ergo: verbi gratia **S** *5* hic] is **X** *6* secuerit] secuit **P** *6* stelle locus] locus stelle **L** *7* planisperio] *corr. ex* plano **L** *8* isdemque] hiisdem *corr. ex* fidemque **K**, hiis igitur **DES**, hisdem **W** *8* et] *om.* **B**

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9 hunc gradum] cum quo mediat stella celum *marg.* **E** (*cf.* **Ar.**)

distantes orizonti, quos arabes pontes nominant.

< 7^2 >

E 93v marg., F 218v marg.

Alia translatio, dixit meslem: quando facies circulum equidistantem circulo signorum, erit digrediens a circulo signorum cum quantitate latitudinis stelle. Et prohice arcum qui transit per polum signorum, qui est *K* in propositione premissa huic, et per gradum stelle in orbe signorum: et secat orbem signorum in duo media et equatorem.

< 8^1 >

A 160vb, B 12r, P 112r–v; in N the relevant folio is missing

Dixit maslem: et non declaravit quod centra non sunt super unum punctum nisi postquam ostendit lineam *MC* fore longiorem linea *LN*. Et iam declaratum est illud proprius isto, et illud est quoniam reiterabimus figuram et protrahemus a puncto *N* lineam que sit perpendicularis super lineam *DL*, que est *NY*. Et manifestum est quod *DN* est brevior *DC* et *DL* brevior *DM*. Abscidamus ergo de *DC* lineam equalem linee *DN*, que est *DT*, et de linea *DM* equalem *DL*, que est *DZ*: fiet igitur triangulus *DTZ* equalis triangulo *DNL*; et cadet perpendicularis *TO* sicut casus perpendicularis *NY*. Et erit angulus *DNL* equalis angulo *DTZ*. Et quia linea *DC* fuit longior linea *DN*, erit angulus *DNC* maior angulo *DCN*; angulus igitur *DCM* reliquus maior est angulo *DNL*, qui est equalis angulo *DTZ*. Cum ergo protraxerimus a puncto *T* lineam equidistantem linee *MC*, cadet ergo extremitas eius

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9 orizonti] orizontis **C** *9* pontes] poñtes **B**, ponentes **C**, potesim **P** **8^1:** *1* centra non] centrum **P** *2* nisi] ubi **P** *3 LN*] *LM* **MSS** *3* isto] et et illud proprius isto *add.* **P** *7 DC* et *DL* brevior] *supra* **A** *7* Abscidamus] abscidimus **P** *9 DTZ*] *DLZ* **B** *10 NY*] *KY* **P** *13 DNL*] *DLN* **B**

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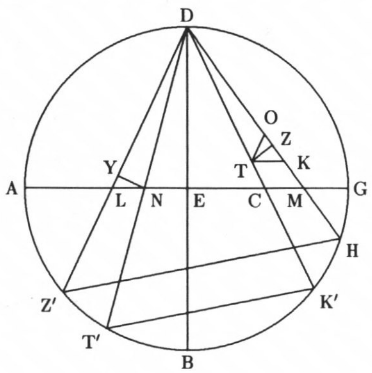
9 quos Arabes pontes nominant] quorum verticales(a) equidistantes(b) circulo recto *add.* **KCDXESWL** [(a)circuli *marg.* **K**, *add.* **CXW**; id est paralleli transeuntes per cenit capitis circuli *add.* **KDESWL** (circuli *del.* **K**, *om.* **W**); id est paralleli ducti ex vertice capitum tanquam centro sunt horizonti ut *add.* **X**; (b)sunt *add.* **KDESW**]; et sunt almucanteraz *marg.* **E** (*cf.* **Ar.**)

altera sub puncto Z, que est linea TK. Proportio igitur CD ad TD est sicut proportio MC ad TK. Longior igitur est CM linea TK, et TK est longior TZ: ergo MC est longior TZ, que est equalis NL. Et si non protrahamus perpendicularem, ostendemus illud per hoc quod dicemus quod angulus DZT, qui est equalis angulo DLN, est accutus; linea igitur KT longior existit linea ZT. Scias. Et ex hac quidem figura declaratur quod sectiones, quas dividunt linee recte protracte a puncto D in linea GA super arcus equales, erunt in linea GA inequales et quanto plus appropinquat centro sit brevior quam cum elongatur ab eo.



[Fig. 3]

< 8^2 >

E 94r marg., F 218v marg.

Dixit meslem: non declaravit quod centra non sint super unum punctum nisi postquam ostendit quod linea MC longior est NL. Et iam explanabitur hoc facilius quia protraham a puncto N perpendicularem super lineam DL, et est NO. Et declarabitur quod DN brevior est quam DC et DL brevior quam

16 igitur] lineam equidistantem linee MC cadet ergo extremitas eius altera sub puncto *add. et del. per "va ... cat"* **P** *17* TD] DT **B** *17* sicut] fuit **P** *17* MC] MT **P** *19* NL] ML **B** *19* protrahamus] trahamus **B** *20* ostendemus] ostendamus **P** *20* dicemus] diximus **P** *21* angulus DZT] angulo DZC **P** *23* quod] omnes *add.* **B** *23* sectiones] *supra* **B** *24* erunt] erit **P** *25* appropinquat] propinquat **B** *25* centro] mare **P** *26* elongatur] elongantur **MSS**

DM. Et secabitur in linea DC equalis linee DN, que sit DT, et de linea DM equalis DL, que sit DZ: fiet ergo triangulus DTZ equalis triangulo DNL; et ut cecidit perpendicularis NO cadat etiam perpendicularis TV, et erit angulus DNL equalis angulo DTZ. Et quia lineata est DC longior linea DN, erit angulus DNC maior angulo DCN; ergo angulus DCM remanet maior angulo DNL, qui equatur angulo DTZ. Quando igitur ducetur a puncto T equidistans linee MC, cadet terminus alter sub puncto Z, et est linea TK. Ergo proportio CD ad TD ut proportio MC ad TK; est ergo longior quam TK, et TK longior quam TZ: ergo MC longior quam TZ, cui equatur NL. Et quamvis non protrahamus perpendicularem, etiam declarabitur hoc ut dicam quod angulus DZT, qui equatur angulo DLN, est acutus; ergo linea KT longior est linea ZT. Et ex hac figura declaratur quod sectiones, quas secant linee recte egredientes a puncto D in linea GA super arcus equales, erunt in eadem preter equalitatem et quod omnes propinquiores centro longiores erunt remotioribus ab eo.

< 9^1 >

A 159v marg., B 10r marg. (illeg. in our copy), N 61r marg. (some words caught in the binding are illegible), not present in P. The section corresponds to Arabic lines 108–117

In alio, dixit maslem: et ex complemento huius questionis est ut protrahamus EQ in rectitudinem et DT in rectitudinem donec concurrant in puncto O; et probabimus quod circulus qui transit per punctum C et punctum Y transit etiam per punctum O, secundum quod probatum est in questione que est ante istam in positione circuli equidistantis circulo orbis signorum ex quo nichil cadit in circulo occulto. Compleamus igitur ei in hac questione omnes conditiones que complete sunt ei in precedenti. Cum ergo voluerimus illud, continuabimus D cum Z et protrahemus ipsam secundum rectitudinem: perveniet ergo ad punctum Q, secundum quod pervenit DH ad

8 cecidit] cedit **F** *21* linea] lineam **MSS** 9^1: *1* et] *om.* **N** *2* DT] *corr. marg.* ex D (?) **A** *9* ergo] *supra* **A**; nota *in textu* **A**

23 longiores] breviores **Ar.** *et versio* **AB** punctum N. Et continuabo D cum K et protraham ipsam in rectitudinem usque ad punctum R, et similiter protraham lineam ZH in rectitudinem usque ad punctum P. Linea ergo ON iam divisa est secundum divisionem linee PH secundum proportionem propter equidistantiam linee PH linee ON. Et protraham a puncto T lineam equidistantem ZH, que est linea TX; arcus igitur DX equatur arcui DT .

< 9^2 >

E 94r marg., **F** 219r marg. The section corresponds to Arabic lines 108–114

Alia translatio, dixit meslem: de complemento huius propositionis est quod producam lineas EQ et DT directe donec concurrant super punctum O; et demonstrabo quod circulus qui transit per notam C et per punctum Y transit etiam per notam O, secundum quod probatum est in propositione antecedente de ponendo equidistante circulo orbis signorum de quo non cadit aliquid intra circulum semper occultum. Cum ergo voluero perficere quod promisi, producam DZ usque ad punctum Q.

< 9^3 >

A 159va–160ra, **B** 10r–v, **N** 61r–v, **K** p. 181–182, **C** 216rb–va, **D** 9v, **X** 23r–v, **E** 94r–v, **M** 18v–19r, **S** 70v, **W** 9r, **L** 55v–56r (variants of **L** not normally noted in the apparatus: *equus* almost always for *equalis* until the last three sentences), **P** 112v–113v. The section begins from Arabic line 113

Deinde argumentum quod maslem subiungit addens: producimus lineam DZ in directum quoad in punctum Q necessario perveniat quemadmodum et DH in punctum N pervenit, ut quemadmodum in supradatis descriptionibus constat. Sicut circulus cuius diametros ZH circa lineam QN describitur, sic

12 ipsam] secundum *add.* **AN** *18 DT*] *hic desinit versio* 9^2: *6* orbis] orbi **E**, illeg. **F** *9 Q*] *hic desinit versio* 9^3: *1* Deinde] Demum **DE**, hic **L**; quidem *add.* **KDEW** *1* quod] *om.* **KDEWL**, quid **MS** *1* maslem subiungit] subiungit maslem **KWL** *1* addens] addentes **ANCS**, addñtes *corr. ex* redñtes **B**, dicens **KWL**; subiungit addens: addens subiungit **P** *2* producimus] producemus **B**, *om.* **S** *2* lineam] *om.* **S** *2* quoad] quousque ad **DE**, quod **P** *2* in] *om.* **CDEXMSWP** *2* punctum] puncto **B** *3* perveniat] proveniat **DE** *3* quemadmodum] sicut **L** *3* et] *om.* **B** *4* quemadmodum] sicut **L** *4* in] ut **B**, *om.* **XP** *4* supradatis] supradictis **BXMSP** *4* constat] constant **S** *4* sicut] sit **XM** *5* sic] sicut **XMP** circulus cuius diametros *TKL* describi possit circa lineam *OC*. Aplicet itaque *D* cum *K* eatque in directum usque ad punctum *R*; sicque *ZH* in directum usque ad punctum *P*; procedat et a puncto *T* in punctum *X* linea equidistans linee *HP*. Divisa est igitur linea *ON* ad similitudinem proportionis partium equidistantis sibi *HP*. Quoniam angulus *DTX* equalis est angulo *DLT*, angulus vero *DTX* est equalis angulo *DPH*, erit angulus *DLT* equalis angulo *DPH*: sunt itaque puncta *L S T P* super circumferentiam circuli locata. Unde quanta *SK* in *KP* ducta tanta *KT* in *KL*, extitit autem quanta *KT* in *KL* tanta *KZ* in *KH*. Equalis est ergo *KZ* in *KH* ducta ei quod *KS* ex *KP* producit. Unde ad eundem modum quanta *QR*

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6 ZH circa ... diametros] *marg.* **S** *6 TKL*] *THL* **P** *6* possit] posset **C** *6 OC*] *EC* **D** *7* itaque] igitur **S** *7* cum] in **N** *7 D* cum *K* eatque] *DM KC* atque **P** *8 R*] *illeg.* **K**, *K* **CML** *8* sicque] sitque **BW** *8 ZH*] *HZ* **X**, *ZB* **L** *8* ad] in **L** *8 P*] *B* **B**; sicque *ZH* ... *P*: *marg.* **K** *9* et] *om.* **EMS** *9* puncto] *seq. ras.* **L** *9 X*] *Z* **S** *9* linee] linea **L** *10* Divisa] itaque *add.* **S** *10* est] *om.* **KDXEMWL** *10* igitur] itaque **DE**, ergo **XP** *10* est igitur linea] igitur linea est **C** *10 ON*] *TN* **D**, *NCO* **X**, *CN* **MS** *10* ad] *supra* **A** *10* linea *ON* ad similitudinem] in dissimilitudinem **P** *10* proportionis] proportione **A**; similitudinem proportionis: proportiones similitudinis **C** *11* equidistantis] linee *supra* **E** *11* sibi] igitur **D** *11 HP*] *HD* **S** *11 DTX*] *DCX* **BDW**, *DTZ* **M** *11* est] *supra* **K** *12 DLT*] *DTL* **B**, *DLC* **W** *12 DTX*] *DCX* **DW**, *DTZ* **M**; *seq. ras.* **S** *12* est] *om.* **CXEM** *12* est equalis] equalis est **AKW**, equus est **L** *13 DLT*] *DTL* **M** *13 DPH*] erit angulus ... *DPH* *om.* **ABNSP** *14 T P*] *om.* **B**, *P T* **P**; puncta *add. et del.* **B** *14* super] *om.* **C** *14* circumferentiam] diametrum **M** *14* Unde] ut **C** *14 SK*] *SB* **M** *15* ducta] producta **C** *15 KT*] *corr. ex KC* **K**, *TK* **X**, *T* **M** *15 KL*] i *add.* **M** *15* extitit] existit **BX**, sint **C**, exstitit **E** *16* tanta] vel in (?) *add.* **A** *16* KH^1] ducta *add.* **KWL** *16* Equalis] qualis **L** *16* est] *om.* **M** *16* KZ^2] *KT* **L** *16* equalis est ... *KH*] equalis est ergo *KZ* (?) *marg.* **B**; *KH*: *HK* **P** *16* ducta] *om.* **C** *16* ei] *om.* **X** *17* ex] in **CDES** *17* tanta *KT* in *KL* ... producit] *marg.* **C**; producit: producto **D** *17 QR*] *RQ* **X**, *corr. ex KR* **E**

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9 HP] *HZ* **Ar.**; et linea *DLC* secet lineam *HZ* in puncto *S* *add.* **X** *12* Quoniam angulus ... *DLT*] Quoniam arcus *DX* equatur arcui *DT* *marg.* **E** (*cf.* **Ar**) *12 DPH*] propter equidistantiam *supra* **E** (*cf.* **Ar.**) *14* locata] hoc probabis compleando punctum(a) *T* cum puncto *S* et proveniet triangulus *PTS*, circa(b) quem describes(c) circulum, et per impossibile probabis ipsum necessario transire etiam per punctum *L* *marg.* **AN** [(a)probabis ... punctum: *corr. ex* compleando punctum probabis **A** (b)circa: circam **A** (c)describes: describens **N**] in RN tanta OR in RC. Applicet itaque R cum Y, eritque triangulus REY similis triangulo KFM, cum et angulus aput F equalis sit angulo aput E et linee eos angulos continentes proportionales. Erunt igitur et reliqui eorum anguli equales: ut cum rectus sit angulus MKF, et angulum ERY rectum esse consequens sit. Equalis est ergo QR in RN linee RY in seipsam ducte. Extitit autem QR in RN equalis OR in RC: equalis est igitur OR in RC linee YR in seipsam ducte. Que cum perpendicularis sit linee CO, puncta C Y O super circumferentiam circuli esse consequens est.

Ex his manifestum est quod in spera dum super idem sint centrum equidistans recto et equidistans zodiaco, medius medium

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18 QR in RN tanta OR in RC] QK in KN tanta OK in KC **KW** [KOC **W**]; KR in KN tanta OR in KC **M** 18 Applicet] applicetur **KWL**, applicatur **DE**, applices **M**, applicentur **S** 18 itaque] ergo **KW**, igitur **DES** 18 R] K **KCM** 18 R cum Y] $KECY$ **W**, RCY **P** 19 REY] KEY **KCDW**, K ei **M** 19 KFM] *corr. ex* KSM ? **S** 20 aput F] APF **P** 20 equalis sit] sit equalis **KCDESWL** 20 aput E] APE **P** 20 linee] apud *add.* **DE** 21 proportionales] erunt *add.* **X** 21 igitur] ergo **X** 21 et] *om.* **BKWL** 22 ut] unde **E** 22 rectus] *corr. ex* rectas **N** 22 angulus] *om.* **C** 22 ERY] EKY **KCW**, ETY **P** 23 consequens] *seq. ras.* **E** 23 sit] est **KXMW**, *om.* **C**, *corr. ex* est **S**; consequens sit: constet **L** 23 est] *om.* **X** 23 QR in RN] QK [R *supra* QR **W**] in KN **KW**, CR in RO **X**, QK in KQ **M** 23 RY] KY **C**, KZ **M** 24 seipsam] seipsa **X**, se **L** 24 Extitit] exstitit **B**, *corr. ex* estitit **N**, existit **S** 24 QR in RN linee ... RN] *marg.* **S**; QR in RN *add. in textu* **S**; linee *add. et del. supra* **S**; RN: KN **W** 24 RC] TR **P** 25 seipsam] se **KW** 25 Extitit autem ... ducte] *marg.* **K**, *om.* **CDXEM** 25 Que] *om.* **S** 26 perpendicularis sit] sit perpendicularis **C** 26 linee] *om.* **C** 26 puncta] igitur *add.* **S** 26 C Y O] Y C O **ABNCDXEMSP**; esse *supra* **K**; esse *add.* **W** 27 esse] *om.* **BKMSWL** 28 manifestum] palam **XM** 28 est] *om.* **A** 28 super] semper **LP** 28 sint] sunt **BKW**, *om.* **XM** 29 recto] *supra* **A**, recte **P** 29 et] *om.* **NP** 29 et equidistans] *marg.* **A**

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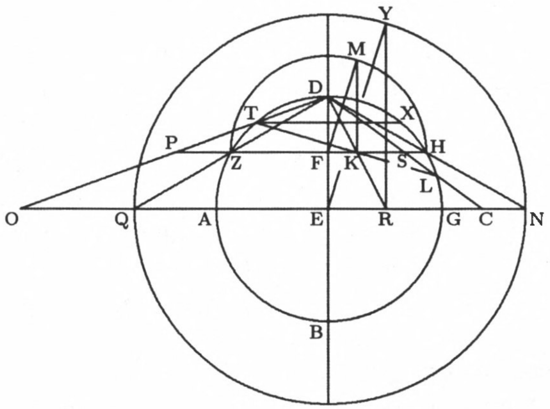
18 Unde ad ... RC] Cum enim trigono (?) NKD et HD [H *supra*] sint equales, cum etiam trigono (?) AKD sunt similes, erit proportio DR ad DK, que est NR ad HK et que est KQ [*lege* RQ] ad KZ; et sic proportio RN ad HK est proportio KQ [*lege* RQ] ad KZ; ergo permutatim proportio RN ad RQ est HK ad KG [*lege* KZ]. Eodem modo constet quod proportio CR ad RO est proportio SK ad KP. Quare (?) quod sit ex HK in KZ est illud quod sit ex SK in KP. Ergo proportio HK ad SK est proportio PK ad ZH; ergo proportio NR ad CR est proportio OR ad QR. Ergo *marg.* **C** 21 proportionales] quia que est proportio NR ad RQ ea est HK ad KZ; ergo coniunctim que est NQ ad RQ ea est HZ ad KZ; ergo que est EQ ad RQ ea est FZ ad KZ; ergo que EQ ad ER ea est FZ ad FK ex v^to euclidis *marg.* **C**; Quia proportio RE ad FK ut proportio ED ad FD propter equidistantiam, quare ut proportio EQ ad FZ *marg.* **E** (*cf.* **Ar.**) 22 Erunt igitur ... MKF] *non in* **Ar.** 25 equalis est ... ducte] et iam fuit QR in RN ut ductus RO in RC; ergo ductus RO in RC equatur ductui RY in se *marg.* **E** (*cf.* **Ar.**)



[Fig. 4]

secat. Quod quoniam planicies ferre non potest, descriptione quam maslem ad id monstrandum hic interponit nos supersedemus, ne quid preter ptholomaice descriptionis nutum ut minus cavemus plus apponamus, presertim dum nulla necessitas cogat. Quod tamen in ipsis eius descriptionibus, qua locus exigit mutatione, maslem non negligimus; nec enim desperet quisquam

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30 ferre] facere **DES**; tes *add.* **M** *30* descriptione] descriptioni **BC**, descriptionem **KDESW** *31* maslem] maslen **DE** *31* monstrandum] demonstrandum **MW** *31* hic] *om.* **B** *31* interponit] unde *add.* **B** *31* nos] *om.* **M** *32* quid] quidem **C** *32* preter] *om.* **DS** *32* ptholomaice] thpolomaice **N**, ptolomaice **BX**, tholomaice **CE**, tolomaice **M**, tholomaeice **S** *32* nutum] numerum **ABNKCDEWLP**, intentum **X**, minuerim **S** *33* cavemus] caveamus **C** *33* dum] cum **KXEWL**, neque **S** *33* necessitas cogat] cogit necessitas **KWL**, cogat necessitas **CDES** *34* eius descriptionibus] descriptionibus eius **XM** *34* qua] que **BSL** *35* mutatione] imitatione **BNKDXMW**, ? **AL**, mutationem **C**, immutare **S** *35* maslem non] non maslen **DES** *35* negligimus] negligemus **P** *35* Nec enim] neque **B** *35* quisquam] aliquis **KWL**

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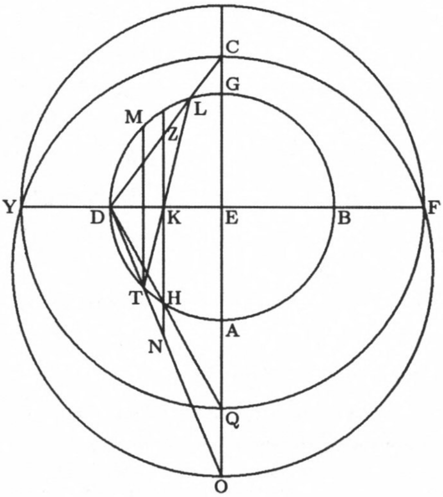
30 potest] Nec miratur cum omnis circulus per polum latentem transiens per lineam rectam representetur. Si enim [*corr. ex* sin] circuli talis planicies et planicies equinoctialis quantuslibet intelligantur produci donec concurrant, omnis communis sectio linea, que sit *BY*, et hoc in plano sita necessario suscipit potentia omne punctum circuli transeuntis per polum *D*. Si enim, ut mos est, in animo a *D* ad punctum datum in circulo lineam dirigas, cum et polus *D* et punctum illud sit in eius circuli plano, erit linea hec necessario in eodem plano. Quare si cum plano producta intelligatur necessario in punctum aliquod communis sectionis, scilicet linee *BY*, incidet. Et hoc est quod ait Ptolomeus i^{v} representari n^{v} incidere *add.* **L** quin nos quoque et ea que maslem interponit et ex nobis ipsis quam plurima eque rationabiliter, ut illi visum est, interserere possemus. Nisi auctorem ipsum ut decet castigare, sequi mallemus veriti ne immoderata evagandi libertas nimie benivolentie vicium incurreret.

< 10^1 >

A 161ra–b, **B** 12r–v (for *equator diei* **B** often has *equatio diei*, which will not be noted in the apparatus), **N** 61v, **P** 113r

Et ex sermone eius etiam est: verumtamen complebo quod oportet compleri in hac questione, quod est quia nos declarabimus quod hic circulus equidistans circulo orbis signorum dividet etiam circulum equidistantem circulo equatoris diei, quem dividit in spera corporali in duo media, in hac figura. Probatio eius: quoniam ponemus circulum meridiei circulum $ABGD$ et polum occultum punctum D et diametron circuli equidistantis circulo signorum TKL. Et continuabo D cum L et producam ipsam usque ad C, et D cum T et producam ipsam usque ad O, deinde protraham a puncto K lineam equidistantem diametro circuli equatoris diei, que est linea AG, et est linea $NHKZ$, et continuabo D cum H et protraham ipsam in rectitudinem usque ad Q. Sicut precessit, quod circulus qui signatur secundum longitudinem EQ est circulus equidistans circulo equatoris diei cuius longitudo ab eo est arcus AH, et est circulus quem dividit circulus equidistans orbi signorum in spera super lineam meridiei. Deinde protraham lineam EB usque ad F et sit EF equalis EQ. Manifestum est igitur quod circulus qui transit per punctum C et O et Y transibit per

36 quin] quoniam **B** *36* quisquam quin nos] qui quam quinos **P** *36* que] quod **N**, quem **C** *36* maslem] maslen **DE** *36* et] etiam **X** *37* quam] *corr. ex* tanquam **K** *37* eque] equa **NCP** *38* auctorem] actorem **K** *38* decet] docet **M** *38* castigare] castigate **KEW** *39* sequi mallemus] valeamus **BS**; mallemus: volumus **P** *39* veriti] verito (?) **M** *39* evagandi] vagandi **C**, evagando **DEM** *39* benivolentie] benevolentie **BX** *40* incurreret] incurret **B**; vicium incurreret: mereretur **P** 10^1: *1* sermone] sevatute ? **P** *1* etiam] Z **P** *3* hic] *seq. ras.* **N** *4* etiam] *om.* **B** *8* circulo signorum] signorum circulo **P** *10* et D cum ... O] *om.* **P** *11* diei] die **P** *12* $NHKZ$] $NBKZ$ **B** *12* D cum H] DCH **A**, $D\ CH$ **N** *12* ipsam] eam **B** *13* in rectitudinem] *om.* **B**; in directione *marg.* **B** *14* equidistans] in *add.* **P** *15* ab eo] $ABRO$ **P** *19* et O et Y] RO et TY **P** *19* transibit] transit **P** punctum F. Protraham ergo a puncto T lineam equidistantem linee KH, que est TM. Angulus igitur DTM equalis est angulo DLT, quoniam sunt super duos arcus equales; et angulus DTM est equalis angulo DNK propter equidistantiam: angulus igitur DNK est equalis angulo DLT. Puncta ergo L Z T N super circumferentiam circuli sunt collocata. Multiplicatio ergo KN in KZ est equalis KT in KL, et KT in KL est equalis KH in seipsam; ergo KN in KZ est equalis KH in seipsam. Et iam divisa fuerat linea OC secundum divisiones linee NZ in proportione: oportet ergo ut sit multiplicatio OE in EC equalis multiplicationi QE in se. Et QE est equalis EF: ergo multiplicatio OE in EC est equalis multiplicationi EF in se. Et illud est quod demonstrare voluimus.



[Fig. 5]

21 TM] *TEM* **P** *21* Angulus igitur] igitur angulus qui est **B** *22 DLT*] *DTL* **P** *23 DNK*] *DMK* **P** *24 DLT*] *DK* **P** *24 L*] et *add.* **P** *26* ergo] igitur **N** *26* est equalis] inequalis **P** *27 KH*] *LEH* (?) **N** *27 KT* in *KL* et ... equalis] *om.* **P** *28 KT* in *KL* et *KT* ... seipsam] *KH* [*corr.*] in seipsam *in textu* **B**, *KT* in *KL* et *K*[*T* in] *KL* est equalis *KH* [in se]ipsam, ergo *KN* in *K*[*Z*] equalis *KH* in seipsam *marg.* **B** *28* iam] causa **P** *28* fuerat] fuerit **B** *28* linea] *om.* **B** *28 OC*] et **P** *29 OE*] *OC* **P** *30* multiplicationi] *EF add. et del.* **A** *30 QE*] *Q* est **B**, que est **NP** *30 QE*] que **B** *32* quod demonstrare voluimus] q. d. v^{i}. **A**, q s v **N**; Et illud ... voluimus *om.* **P**

< 10^2 >

E 94v marg., **F** 219r marg.

Et ut compleam quod oportet compleri in hac propositione, declarabo quod hic circulus equidistans orbi signorum secat etiam parallelum, quem secat in spera in duo media, in hac forma in duo media. Cuius hec ratio quod ponam circulum meridiei $ABGD$ et polum occultum punctum D et diametrum circuli equidistantis circulo signorum TKL. Et ducam DL ad C et DT ad V, et traducam super punctum K lineam equidistantem diametro equatoris, qui est AG, et est linea $NHKZ$, et iungam D cum H et dirigam ad Q. Et iam premisimus quod circulus qui lineatur cum longitudine EQ est circulus equidistans quem secat circulus equidistans circulo signorum in spera in duo media super lineam medii diei. Et producam lineam EB ad F, et erit EF equalis EQ. Declarabimus quod circulus transiens per notas C et V et O transit etiam per punctum F. Et traducam super punctum T lineam equidistantem linee KH, et est TM. Ergo angulus DTM equatur angulo DLT, quia ambo sunt super arcus equales; et angulus DTM equatur angulo DNK propter equidistanciam: ergo angulus DNK equatur angulo DLT. Ergo puncta L Z T N sunt super circumferenciam. Ergo ductus KN in KZ equatur KT in KL, et KT in KL equatur KH in se; ergo KN in KZ equatur KH in se. Dividitur ergo linea VC similiter linee NZ in proportione: oportet ergo quod sit ductus VE in EC ut ductus EF in se. Ergo circulus transiens per puncta C O V transit etiam per punctum F; et hoc est. Et secundum similitudinem huius ducam circulos equidistantes orizonti cadentes sub orizontem, nec est differentia inter hec. Et erunt almucanterat cadentes in lamina super astrolabium meridianum.

< 11 >

A 160ra–b, **B** 10v–11r, **N** 62r, **K** p. 182–183, **C** 216va–b, **D** 10r, **X** 24r–v, **E** 94v, **M** 19r, **S** 71r, **W** 9v, **L** 56r–v (frequent use of *equus* for *equalis* is not noted in the apparatus), **P** 113r

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3 quem] que **MSS**

Addit maslem quoniam hec linea recta secat circulum latentem in arcus similes arcubus quos recidit in spera corporea. Quod ut planius constet esto diametros circuli equidistantis recto perpetuo latentis linea $ZFKH$. Eritque circulus descriptus ad distantiam AZ de perpetuo latentibus; fiat itaque super lineam ZH semicirculus eatque a puncto K linea KM equidistans linee ED. Quemadmodum itaque circulus equidistans zodiaco designatus super diametron DL secat in spera circulum latentem ad punctum M in arcus HM et MZ; sic linea BY circulum NYQ in arcus NY et YQ arcubus HM et MZ similes. Cuius argumento applicabit E cum Y et F cum M. Quoniam itaque linea FH equidistans est linee NE, eritque proportio NE ad EC eadem FH ad FK. Sed NE equalis EY sicque FH equalis FM; que ergo proportio EY ad EC eadem MF ad FK, atque angulus YCE rectus sicque angulus MKF: similis est itaque triangulus MFK triangulo YEC; sic igitur et angulus YEC equalis est angulo MFK; unde arcum NY arcui HM. Sicque reliquum reliquo de semicirculis similes esse consequens est. Secat itaque linea BY circulum

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1 Addit] Adde **C** 1 maslem] maslen **E**, *corr. ex* malem **S** 1 quoniam] quantum **XM**, quod **L** 1 secat] *om.* **X** 2 recidit] rescidit **KWL**, recedit **C**, rescindet **X** 3 planius] planicies **L** 3 constet] *supra* **B**, fuit **L** 3 esto] *corr. in* Esto **B** 3 equidistantis] equidistantes **ABNP** 4 perpetuo] semper **L** 4 latentis] latenti **M** 4 linea] *om.* **S** 4 *ZFKH*] *KH* **P** 5 circulus descriptus] descriptus circulus **ABNP** 5 distantiam] *corr. ex* distandtiam **A**; ad distantiam: a distantia **DES** 5 perpetuo] semper **L** 7 linee *ED*] line *DE* **L**; *seq. ras.* **S** 7 Quemadmodum] sicut **L** 7 circulus] vel linea *add.* **L** 8 diametron] *om.* **DE** 9 latentem] occultum *supra* **E**, latantem **K** 9 *HM*] *LHM* (?) **B** 9 sic] sit **P** 10 *NYQ*] *NYA* **L** 11 argumento] augmento **B**, argumentorum **C** 11 *F* cum] sicut **C** 12 eritque] erit que **L** 13 *EC*] *DC* **B**, *EA* **P** 13 equalis] est *add.* **KDESWL** 14 Sicque] sique **X** 14 *FH*] *FB* **D** 14 equalis] est *add.* **D** 14 que] Quia **K**; in *add.* **S** 14 ergo] *supra* **E**, vero **L**; est *add.* **E** 14 *EC*] est *add.* **K** 15 *FK*] *KF* **KCDEW**, *BF* **L**, *FH* **P** 15 atque] et **L**, itaque **P** 15 sicque] sitque **P** 16 *MKF*] *om.* **A** 16 similis] quoque (?) circulus **P** 16 est] *om.* **L** 16 est itaque] itaque est **KW** 16 *MFK*] in *add.* **C** 16 *YEC*] *YCE* **S** 17 sic] sit **P** 17 igitur] ergo **XP** 17 similis est ... et] *marg.* **B** 17 *YEC*] *YET* **BP** 17 similis est ... angulo] *marg.* **A** 17 *MFK*] *MKF* **AC** 17 unde] et *add.* **C** 18 *HM*] *BM* **BKDW** 18 Sicque] Sitque **BP** 18 reliquum] reliqum **B**, residuum **DE**; de *add. et del.* **K** 19 consequens est] est consequens **C**, constat **L**

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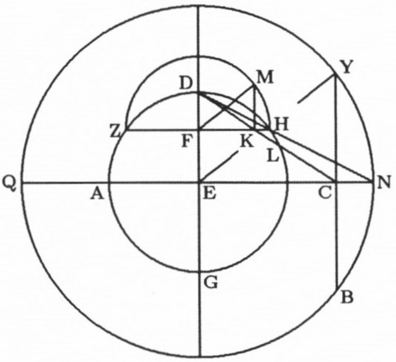
4 ZFKH] et ducam *DHN*; ergo equidistans equatori cum longitudine *EN* lineatur et (?) occultatur semper *marg.* **E** (*cf.* **Ar.**) 5 *AZ*] *EN* **Ar.** 12 *FH*] *ZH* **Ar.**

NYQ in arcus similes arcubus quos circulus equidistans zodiaco de circulo latente resecat in spera corporea. Amplius cum circulus hic per polum latentem transeat, in ea planicie polus ille <non> incidit in cuius partem planicies poli apparentis incipiat; minime tamen usque ad polum illum pervenit. Sic linea *BY*, licet in infinitum protrahatur, numquam linee *GD* concurret.



[Fig. 6]

Ex his manifestum est quod consequens est cum hic circulus equidistans zodiaco per polum circuli recti transiens hunc equidi-

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20 *NYQ*] *AYQ* **P**; e *supra* **P** *21* zodiaco] zodiacos **P** *21* resecat] recesant **L**, *om.* **P** *21* Amplius] *om.* **X** *21* cum] ergo *add.* **X** *22* hic] *om.* **X** *22* transeat] in spera *supra* **E** *22* ea] *supra* **S** *22* ea planicie] eo plano **L** *23* incidit] incidint (?) **A**, recidit **B**; ille incidit *corr. ex* incidit ille **K**, incidit ille **W**; m *add.* **X** *23* cuius] planini *add. et del.* **K** *23* partem] cum *add.* **MSS** (tum **D**) *24* incipiat] incipat **N**; apparentis incipiat: incipiat apparentis **DE** *24* minime] non **L** *24* tamen] cum **XP** *24* pervenit] provenit **ABNML**; illum pervenit: pervenit illum **X** *25* *BY* licet] *BYLZ* **P** *25* linee *GD*] secum **X**; *GD*: *DG* **ESLKW** *26* concurret] *MKF* Similis est itaque triangulus *MFK* triangulo *YET*, sic igitur et angulus *add. et del. per* "*va ... cat*" **B** (*cf. textum l. 16–17*) *27* his] hoc **C** *27* manifestum est ... est] claret **L** *28* recti] *om.* **X**; recta *add. et del.* **C**; circuli recti: recti circuli **L** *28* hunc] hic **X**

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22 planicie] q. d. et non cadit in hac superficie polus occultus cum eminente, quoniam hec superficies, et si in infinitum protendatur, non perveniet ad meridianum polum *marg.* **E** (*cf.* **Ar.**) *27* his] in alio libro non erat hoc *marg.* **AN** stantem recto medium secet, et hunc per zodiaci polum necessario transire.

3. Maslama's Extra-Chapter, Latin versions

a) Version ABNP

A 161r–162r, **B** 12v–14r, **N** 62v, 81r, 81v, **P** 114r–115r

Capitulum quod non est de libro, quod edidit abualcacim maslem filius ameti.

Dixit maslem filius hameti. Iam rememoratus est ptolemeus in hoc libro qualiter describamus circulum orizontis et circulos equidistantes ei, qui sunt almucantarat, et qualiter describamus circulum orbis signorum et circulos equidistantes ei. Et rememoratus est qualiter dividamus circulum orbis signorum per gradus et signa duobus modis, et non rememoratus est divisionem circuli orizontis neque circulorum equidistantium ei qui sunt azumut. Modus autem unus in divisione orbis signorum est ut faciamus circulum equidistantem circulo equatoris diei secundum longitudinem declinationis unius gradus et unius gradus aut secundum longitudinem declinationis duorum graduum et duorum graduum et circumducemus circulos qui transibunt per gradus signorum; et est secundum propin-

29 medium] medio **B** *29* zodiaci] *corr. ex* zodiacum **B** *29* zodiaci polum] polum zodiaci **X** *30* necessario] necessarium est **KW** **Ex.-Ch.**: *1* est] *supra* **A** *1* abualcacim] abualcatim **P** *2* ameti] almeti **B**, hameti **N** *3* hameti] Halmeti **B**, ameti **P** *3* rememoratus] remoratus **B** *4* circulum] oculum **A** *4* circulos] *om.* **B** *5* ei] diei **B**; *seq. ras.* **N** *5* almucantarat] *corr. ex* almucantatarat **A**, almicantorat **B** *6* circulum orizontis ... describamus] *om.* **N**; describamus: describemus **B** *6* circulum] circulos **B**; orizontis et ... circulum *om.* **P** *8* per] et **B** *8* duobus] diebus **P** *8* rememoratus] memoratus **B** *10* azumut] azimut **B** *10* divisione] divisionem **P** *12* longitudinem declinationis] declinationes longitudinem **P**

29 medium secet] intellige medium non per medium sed existentem medium eorum quos equidistantes zodiaco secat *add. et del. inter "va ... cat"* **K**; interpositio sive (?) expositio et non est textus: medium non per medium sed existentem medium eorum quos equidistantes per medium secat *add.* **L**, *addens in marg.* nota **Ex.-Ch.**: *10* qui] *wa-hādhihi 'l-aqsām* **Ar.** *11* circulum equidistantem] **Ar.** *plur.*

quitatem exitus ad actum. Et secundus modus ut protrahamus lineas rectas per centrum circuli equatoris diei, et protrahemus eas ab equatore diei super ascensiones unius gradus et unius gradus de spera recta. Tunc ipse transibunt per gradus orbis signorum et hoc est sanius. Et in eo est tertius modus: qui est quia omnes duo circuli magni in spera cadentes sese secant in duo media. Cum ergo fecerimus super speram circulum tertium magnum secantem se cum eis in loco sectionis eorum, et diviserit quod est inter duos circulos in duo media, et posuerimus a polo eius portionem circuli magni, secabit de illis duobus circulis duos arcus a sectione, et ipsi sunt equales. Et probatio illius est apparens in spera. Quod est quoniam circulus *AHG* est circulus magnus < ... > qui se secat cum istis duobus circulis super duo puncta *A* et *G*, < ... > et iam divisit circulus iste arcum *BD* in duo media in puncto *H* et polus eius est punctum *T* et iam divisit iste polus quod est inter *E* et *Z* etiam in duo media. Deinde ponamus arcum a polo *T*, qui est arcus *TKLM*. Dico ergo quod arcus *AK* est equalis arcui *AM*. Et probatio eius: quoniam unusquisque angulorum *ALK ALM* est rectus, quia punctum *T* est polus circuli *ALHG*, et unusquisque duorum angulorum *KAL MAL* equatur alii, quoniam arcus *DH* est equalis arcui *HB*, et idcirco divisit arcus *AH* angulum *KAM* in duo media, et arcus *AL* est communis, arcus ergo *AM* equatur arcui *AK*. Cuius hec est forma.

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16 exitus] *supra* **B** *16* protrahamus] protrahemus **B** *17* protrahemus] protrahamus **B**, trahemus **P** *18* protrahemus eas ab equatore diei] *marg.* **A** *23* circulum] circulos **B** *23* sectionis] *corr. in* sectionum **N**, sectionum **AB** *25* posuerimus] posuimus **P** *25* eius] per *add.* **P** *26* a sectione] septentrione **P** *27* illius] eius **P** *28* se secat] secat se **BP** *30* in puncto] puncta **P** *31* *T*] *corr. ex G* **B** *31* polus] arcum *BD add.* **B** *31* quod] qui **B** *32* *E*] *supra* **B** *33* ergo] igitur **B** *34* Et] *om.* **P** *35* *ALK*] *ALT* **P** *36* *ALHG*] *ALBG* **N** *36* unusquisque ... unusquisque] unus **B** *36* *KAL*] *corr. supra in ALK* **B**, *KAM* **Ar.** *37* *MAL*] *M supra* **B** *37* *DH*] *AH* **NP** *38* *KAM*] *KM* **P**

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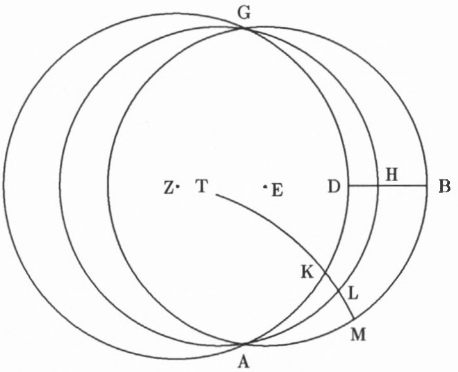
28 *AHG*] *ABG* **Ar.** *28* < ... >] *hic deest* **Ar.** p. 23,5–7, *taqaʿu* ... *ʿaẓīma* *37* *MAL*] est rectus quia punctum *T* est polus circuli *ALHG* et unusquisque duorum angulorum *KAL MAL* *marg.* **B**

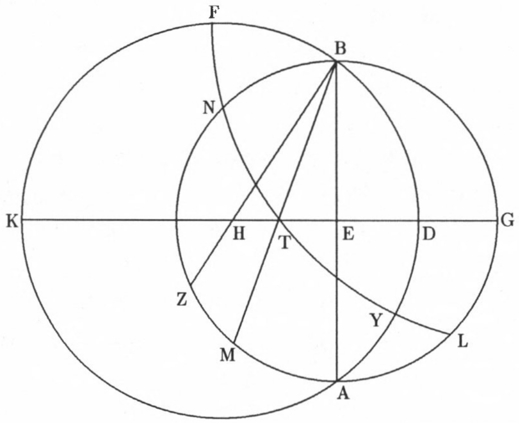


[Fig. 7]

Cum ergo declaratum est istud, circumducamus ergo circulum equatoris diei, qui est circulus ABG, circa centrum E, et circulus orbis signorum sit circulus $ABDK$, et secemus arcum AZ et ponamus eum equalem ei qui est inter tropicum et equatorem diei, et continuemus punctum Z puncto B. Fiet ergo punctum H polus orbis signorum secundum quod declaravit Ptolomeus in hoc libro. Quando ergo diviserimus arcum AZ in duo media in punctum M et continuaverimus B cum M, fiet punctum T polus circulo maiori qui se secat cum circulo equatoris diei in duobus punctis A et B et dividet quod est inter tropicum et equatorem diei in duo media. Cum ergo acceperimus circulo equatoris diei arcum 30 graduum, qui est arcus AL, et a puncto B arcum ei equalem, qui est arcus BN, et protraherimus super istos duos arcus arcum transeuntem per punctum L et N et per polum T, qui est arcus LTF, oportet ut sit arcus AY signum arietis et arcus BF signum libre. Et similiter divides totum circulum per gradum gradum. Et hec est forma illius.

41 circumducamus] circumdamus **B** *45 Z*] *om.* **P** *47* Quando] *corr. ex* Quoniam **B** *47* arcum] arcus **NP** *48* punctum] puncto **BP** *48* continuaverimus] continuabimus **B**, continuavimus **P** *48 B*] *supra* **B** *49* se] *om.* **B** *53 AL*] *AB* **P** *53 B* arcum] *T* arcus **NP** *54* protraherimus] protrahemus **B**, protraximus **P** *55 LTF*] *LIF* **B** *56 AY*] *YA* **P** *57* gradum] et *add.* **P**

45 inter ... diei] *bayna 'l-munqalabayn* **Ar.** *51* inter ... diei] *bayna 'l-munqalabayn* **Ar.**



[Fig. 8]

Opus autem divisionis circuli orizontis in 360 partes ad cognitionem solis, in quacumque hora acceperis considerationem eius, est sicut opus in circulo orbis signorum secundum illos 3 modos. Primus modus est ut scias quanta sit declinatio circuli orizontum ab equatore diei, quod est ut minuas latitudinem civitatis de 90 semper, et quod remanet est illud quod est inter orizontes tuos et circulum equatoris diei. Quod pones loco 24 graduum qui sunt inter duos tropicos et equatorem diei, quasi dicatur declinavit orbis signorum ab equatore diei tantum et tantum quantum provenit unicuique graduum signorum de declinatione. Cum ergo exierit illud tibi, invenies circulos equidistantes equatoris diei secundum illos numeros qui tibi provenerunt. Tunc igitur dividunt isti circuli equidistantes circulum orizontum secundum numerum partium eo-

59 divisionis] *supra* **B** *61* eius, est] eiusdem **B** *64* illud] istud **P** *66* 24] et quia **P** *67* ab equatore diei] *om.* **B**; per equatorem diei **B** *69* de declinatione] declaratione **B** *69* invenies] eveniens **P** *70* equatoris diei] equatori **P** *70* illos] istos **P** *71* qui] *om.* **B** *71* isti] illi duo **B**

60 cognitionem] *samt add.* **Ar.** *63* orizontum] **Ar.** *sing.* *65* orizontes tuos] **Ar.** *sing.* *67* inter ... diei] *bayna 'l-munqalabayn* **Ar.** *72* orizontum] **Ar.** *sing.* *72* numerum] **Ar.** *plur.*

rum; verumtamen exitus huius ad effectum habet in se propinquitatem sicut rememorati sumus ante hoc in interiori parte huius libri. Et modus secundus ut dicamus declinavit orbis signorum ab equatore diei tantum et tantum quanta est ergo ascensio unius gradus unius gradus ex eo in spera recta. Cum ergo computaveris illud, produces lineas rectas transeuntes per centrum circuli equatoris diei et per tempora ascensionum in circulo equatoris diei, tunc ille transibunt de circulo orizontum per numerum partium eorum. Tertius modus est secundum quod precessit in forma predicta, quod est ut ponamus circulum orizontis circulum ADK et polum eius punctum H, ita ut ponamus arcum AZ equalem complemento latitudinis regionis tue; erit ergo punctum H semt capitum in tabula. Et divides quod est inter polum eius et polum equatoris diei in duo media potentia super punctum T, et divides arcum AZ in duo media in puncto M et continuabis B cum M, et secabimus de circulo equatoris arcum quantum voluerimus, qui est arcus AL, et ei est equalis BN; et faciam pertransire arcum equalem arcui LTF. Erit ergo quantitas arcus AY de circulo orizontis equalis quantitati AL de circulo equatoris diei, et equalis ei arcus BF. Et similiter facies per gradum gradum, si poteris facere hoc. Cum ergo completum fuerit tibi illud, protrahes arcum a puncto Y usque ad punctum F transeuntem per punctum H, qui est polus orizontis, et similiter per omnes duas sectiones ex sectionibus diametralibus. Erit ergo sectio circuli orizontis et circulorum ei equidistantium, qui sunt almucatarat, super partes eorum.

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74 sumus] fuimus **BNP** *74* in interiori] ut minori **B**, in maiori **P** *75* libri] *corr. ex* libroi **A** *75* dicamus] quantum *add.* **MSS** *76* tantum] ratum **N** *77* unius] diei *add. et del.* **A** *77* unius gradus] *om.* **B** *78* computaveris] computaverimus **B** *80* et per ... diei] *om.* **P** *80* de circulo] *corr. ex* per circulum **A** *82* quod] que **B** *85* semt] sem^{t} **B**, sont **ANP** *86* eius et polum] *om.* **B** *88* potentia ... media] *om.* **B** *89* secabimus] cum *add. et del.* **N** *91* *LTF*] *LIF* **B**, *LEF* **P** *93* ei] est **B** *93* gradum] *om.* **B**, graduum **P** *94* poteris facere] facere poteris **B** *97* duas sectiones] sectiones duas **A** *97* sectionibus] ecaciobibus ? **B** *97* sectio] eius *add.* **MSS** *99* almucatarat] almucaratat **B**, almucantarat **P**

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81 orizontum] **Ar.** *sing.*

Et cognitio positionis stellarum fixarum in alancabut: est illud ut facias circulum equidistantem circulo orbis signorum cuius longitudo ab eo sit equalis latitudini stellarum et in parte latitudinis. Deinde facies pertransire arcum qui transeat per gradum stelle in orbe signorum et per oppositum eius et per polum orbis signorum: ubi ergo arcus secuerit circulum equidistantem orbi signorum, ibi est locus stelle. Et hic est alius modus in positione eius, qui est ut scias longitudinem stelle ab equatore diei in circulis meridiei et scias gradum cum quo mediat stella celum. Facias pertransire igitur lineam transeuntem per centrum circuli equatoris diei et per gradum mediationis: ubi ergo secuerit linea circulum equidistantem equatori diei descriptum secundum equalitatem longitudinis stelle ab equatore diei, ibi est locus stelle. Et est ei tertius modus, qui est ut scias cum quo gradu ascendit stella ad latitudinem datam de latitudinibus regionis et cum quo gradu occidit ad illam latitudinem. Deinde ponas stellam ascendentem cum illo gradu cum quo ascendit cum portione circuli orizontis facta ad equalitatem illius latitudinis date, et ponas ipsam occidentem cum gradu occidentis: ubi ergo se secuerint circuli orizontum, ibi est locus stelle. Exempli causa in vulture cadente et latitudo climatis data 39 partes, et ascendit in illa latitudine cum 12 gradibus scorpionis; facies ergo cadere arcum de circulo orizontis super 12 gradus de scorpione, sicut arcus BG. Et occidit cum 12 de aquario; facies ergo cadere arcum super 12 de aquario occidentem, qui sit arcus EZH: punctum igitur Z qui

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100 fixarum] est *add.* MSS *100* alancabut] alhaltabut *corr. ex* alhaltabat B *101* facias] facies B *102* longitudo] latitudo P *102* sit] fit AN *104* gradum] gradus P *104* eius] orbis B *106* orbi] orbis NP *106* ibi] ubi B *107* est ut scias] quam vel sumas B; que est ut *supra* B *109* stella] stelle N *109* Facias] facies P *109* pertransire igitur] igitur pertransire B *111* mediationis] scilicet celi *supra* AN *111* secuerit linea] linea secuerit N *111* circulum equidistantem] equidistantem circulum B *112* equalitatem] quantitatem P *116* ascendentem] cum illa *add. et del.* A *118* equalitatem] equatorem P *118* illius] huius B *119* ergo] igitur B *119* secuerint] secuerit B *122* 12] uno et P *123* 12] 1 et P *124* occidit] occidet AB *124* occidet cum 12] occidetium N; occidit cum 12: occidentium P *124* facies] facias AB *124* 12] uno et P

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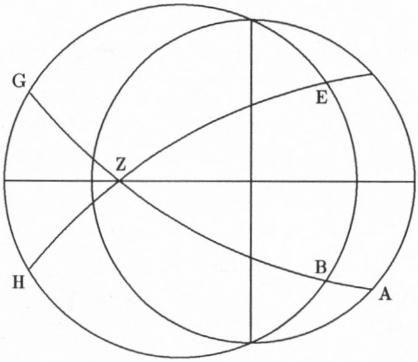
102 stellarum] Ar. *sing.* *108* circulis] Ar. *sing.* *115* regionis] Ar. *plur.* *117* facta] *leg.* facti (*sic* Ar.)

est punctum sectionis est locus stelle in tabula. Verumtamen exemplificavimus hoc per numeros secundum propinquitatem non secundum veritatem, quoniam per illos numeros ascendit et per eos occidit. Et secundum hoc perficietur tibi quod volueris de scientia tabularum. Et laus sit deo creatori gentium.



[Fig. 9]

b) Version **EF**

E 55v–56r, **F** 198r–v

Sectio que non est de libro, quam dixit meslem, usque ad primam sectionem quam ptholomeus

<I>am memoravit ptholomeus in hoc libro quomodo lineentur orizontes et equidistantes illis, qui sunt almucantarat, et quomodo lineetur circulus signorum et equidistantes illi. Dixit etiam quomodo dividatur orbis signorum in gradus et signa secundum duas formas, et non memoravit divisiones orizon-

127 exemplificavimus] exemplificabimus **B** *127* propinquitatem] et *add.* **B** *128* quoniam] non *add.* **ABN** *131* Et laus ... gentium] a-m-e-n. m-e-n. Deo Agamus - Gracias -Summo creatori - omnium Uero - incomutabili - ac etiam pye sue matri - **B**, Finis laus deo **P** vers. **EF**: *2* Sectio que ... ptholomeus] *marg.* **E** *4* almucantarat] *corr. ex* almucanterat **E**, almucantharat **F**

4 orizontes] **Ar.** *sing.* *4* illis] **Ar.** *sing.* *7* divisiones] **Ar.** *sing.*

tum et circulorum equidistantium illis, que sunt azumut, id est linee verticales. Una ergo divisionum orbis signorum est ut faciamus circulos equidistantes equatori cum longitudine graduum et signorum in declinatione, et transibunt per gradus signorum; et hoc fere ut perveniat ad effectum. Altera ut tranducamus lineas rectas per centrum equatoris et elevationes graduum in spera recta, que transibunt per gradus; et hoc est rectius. Ad hoc forma tertia: quia omnes duo maiores spere circuli se dividunt per equalia. Quando ergo fecerimus in spera circulum tertium maiorem secantem eos super eorum sectiones et dividentem quod est inter illos in duo media, a cuius polo descendat pars circuli maioris, ipsa separabit a circulis duos arcus equales. Cuius hec est ratio apparens in spera: ut sit circulus ABG maior secans maiorem ADG, et sint B et D super quartam; et faciemus inter illos tertium et maiorem qui dividat quod est inter illos in duo media et ipsos etiam super A et G, qui sit AHG, et polus eius punctum T, qui est in medio polorum reliquorum; et ducemus etiam arcum $TKLM$. Dico ergo quod arcus AK equatur arcui AM. Quia angulus ALK equatur angulo ALM recto quia T polus est circuli $ALHG$, et angulus KAL equatur angulo LAM quia arcus DH equatur arcui HB, dividit ergo arcus AH angulum MAK in duo media. Et arcus AL sit communis; reliquum ergo patet.

<E>t postquam patet hoc, si duxerimus circulum equatoris ABG circa centrum E et circulum signorum ADB, et secuerimus arcum AZ equalem declinationi et duxerimus lineam BHZ, erit H polus signorum secundum quod dixit ptholomeus in hoc libro. Cum ergo diviserimus arcum AZ in duo media super M et duxerimus BTM, fiet T polus circuli maioris secantis equatorem super B et A et secabit declinationem per medium. Quando ergo accipiemus ab equatore arcum xxx graduum, qui sit AL, et a puncto B arcum equalem BN, et lineaverimus arcum $LOTNF$, oportet quod sit AO signum arietis et FB

12 perveniat] perveniant **E** *40* arcum] arcis **E**

8 orizontum] **Ar.** *sing.* *8* illis] **Ar.** *sing.* *17* sectiones] **Ar.** *sing.*

signum libre; et secundum hoc dividemus totum gradatim circulum. Et est hec huius forma.

<P>er divisiones autem orizontis in 360 partes scitur locus solis, quandocumque accipitur eius comparatio; fit autem in hiis, sicut fit in circulo signorum, tribus modis. Quorum primus est ut sciamus quantitatem declinationis orizontis ab equatore, et illud ut minuamus latitudinem regionis de xc semper: quod ergo remanet est id quod est inter orizonta et equatorem. Et ponemus illud loco xxiiii graduum qui sunt distantia tropicorum, ac si dicatur declinat ab equatore zodiacus sic aut sic quantum accidit unicuique de gradibus signorum de declinatione. Cum ergo provenerit tibi hoc, educes circulos equidistantes equatori secundum numerum qui provenerit tibi: divident isti circuli equidistantes orizontem secundum numerum partium eius. Ex hoc ergo provenies ad effectum in eo fere secundum quod memoratum est in hoc libro. Secundus modus est quod dicas declinat orbis signorum ab equatore sic aut sic quantum elevatur cum gradibus in spera recta. Cum ergo numeraveris hoc, educes lineas rectas transeuntes per centrum equatoris et per tempora elevationum in equatore: et educentur ad orizontes super numerum partium eorum. Modus tertius est secundum quod prediximus in forma memorata ante, et hoc est ut faciamus circulum orizontis ADK, et polus eius H, secundum quod fiat arcus AZ equalis complemento latitudinis regionis tue; erit ergo punctum H summitas capitum in lamina. Et dividam quod est inter polum eius et polum equatoris in duo media potentialiter super punctum T: dividetur ergo AZ per equalia super M; et ducam BM. Et secabo ab equatore arcum quantum voluero, et est AL, et equalem illi, BN, et ducam arcum LTF. Erit ergo quantitas arcus AO de orizonte ut quantitas arcus AL de equatore, et equalis est arcus BF. Et secundum hoc facies per singulos gradus, si po-

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54 **isti circuli] istis circulis MSS** *57* **declinat] declinationem MSS** *57* **sic] et** ***add.*** **E**

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43 **divisiones] Ar.** ***sing.*** *53* **numerum] Ar.** ***plur.*** *54* **numerum] Ar.** ***plur.*** *61* **orizontes] Ar.** ***sing.***

teris. Cum ergo feceris hoc, prohicies arcus a puncto O ad punctum F transeuntes per punctum H, qui est polus orizontis, et similiter per singulas sectiones. Erit ergo divisio orizontis et equidistantium ei, qui sunt almucantarat, secundum partes eorum.

Et scientia positionis stellarum fixarum in aranea est ut fiat circulus equidistans circulo signorum cuius longitudo sit ab eo sicut latitudo stelle et in eandem partem, et duces per gradum stelle in orbe signorum et oppositum eius et polum ipsius arcum: ubi ergo secabit equidistantem, erit positio stelle. Est etiam modus alter ut scias longitudinem stelle ab equatore in circulo meridiei et scias gradum cum quo mediat ipsa celum. Transeat ergo linea per centrum equatoris et per gradum cum quo mediat ipsa celum: et ubi secuerit hec linea circulum equidistantem equatori lineatum cum longitudine stelle ab equatore, est positio stelle. Et est forma tertia ut scias cum quo gradu elevatur stella in latitudine nota de latitudinibus regionum et cum quo occidit in eadem latitudine. Et ponas stellam elevari cum eodem gradu cum quo elevatur ipsa per partem orizontis facti super illam latitudinem notam, et ponas casum gradus occasus: ubi ergo secabuntur partes horizontis, est positio stelle. Cuius exemplum est in vulture cadente et latitudo nota climatis xxxix partes. Et elevatur in hac latitudine cum xii gradu scorpii, ponatur ergo arcus orizontis super xii gradum scorpii, ut arcus BG; occidit etiam cum xii gradu aquarii, cadat ergo arcus orizontis super xii gradum aquarii occidentem, ut arcus EZH: est ergo sectio Z positio stelle in lamina – fere non vere – quia cum eodem numero elevatur cum quo occidit. Et secundum hoc fit artificium laminarum.

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86 celum] Transeat ergo *add.* **E** *90* ponas] i *add. et del.* **E**

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73 arcus] **Ar.** *sing.* *74* transeuntes] **Ar.** *sing.* *78* positionis] **Ar.** *plur.*

c) Notes to the Arabic text

Annotations to the Arabic text as edited by Vernet and Catalá, pp. 22-26 (according to page and line numbers; changes in the points of the prefix letters of verbal imperfects are only noted when required by the subject of a sentence).

22,4 قسمة: قسمه 7 تجوز: يجوز 8 دائرة: *del.* MS; سعدر ط: بتقريب *add. marg.* MS

23,1 تتقاطع: يتقاطع 3 الاولين: الاولتين 4 تكونان: يكونان 6 نقطة ز: نقطة ر 10 ه وز: ه ور 14 فقوس ام: فقوس ال, *supra* ام MS 17 قوس از: قوس ار 18 زب: رب (رح, *supra* رب MS); فتصير: قتصير 19 قوس از: قوس ار 20 فتصير: فيصير 21 اخذنا دائرة MS: *leg.* اخذنا في دائرة (*cf.* 24,21)

24,1 وخططنا: وحططنا; ونقطة: وبقطب MS 10 اخرج: خرج MS 11 استقسم: ستقسم MS 14 درجة: درجة درجة MS 17 هو ما MS: *leg.* هو على ما; يقوم MS: *leg.* تقدم; فعل MS: *leg.* قبل 18 دائرة ادك: دائرة ادلج MS 19 قوس از: قوس ار 20 بنصفين...: بنصفين بالقوة MS 21 قوس از: قوس ار; وتصل جم: وتصل بم MS 22 ومثلها بن: ومثلها جن MS

25,2 ومثلها: وميلها MS *false*; قوس جف: قوس بف MS 3 فاذا اكمل: فاذا كمل MS 4 فتكون: فيكون 7 فيكون: يكون MS; تمر: تمر قوسا قوسا تمر MS 16 تقطعه: بقطعه MS (*Lat.* بقطعة) 18 للاقليم: الاقليم MS 20 قوس بح: قوس بحـ MS 21 غاربة: غاربه 22 قوس هدح: قوس هزح MS; فنقطة ز: فنقطة ن MS

4. Maslama's Astrolabe Chapters, Latin version

a) Latin text

E 56r–57v, F 198v–200r. Notice: an unknown Latin author added comments to chapters I–IV, which are printed below in smaller type.

Et hec capitula non pretermittat qui voluerit facere astrolabium que compilavimus de figura sectionis.

I <A>d scientiam extrahendi elevationes signorum in orbe recto: cum volueris, accipies ergo declinationem extremitatis capricorni, quam minues de xc et accipies sinum residui et servabis. Deinde accipies sinum numeri graduum capricorni, qui sunt xxx, et duces in medietatem diametri semper, et divides quod provenerit per id quod servaveras. Quod ergo exierit arcuabis, et hic arcus est elevatio capricorni. Et accipies declinationem finis aquarii, quam minues de xc, et pones residuum sinum, quem servabis; et accipies sinum lx, numeri graduum capricorni et aquarii, quem duces in medietatem diametri, et divides agregatum per reservatum; et quod exierit arcuabis, et hic arcus est elevationes capricorni et aquarii. Et secundum hoc facies de singulis gradibus donec compleatur quarta.

Sit A polus australis, capud arietis B, capricornus CD, declinatio finis capricorni DE, maxima declinatio CF. Igitur inter duos arcus AF et BF duo arcus AE et BC secant se ad punctum D, et quilibet istorum arcuum est arcus maximi circuli in spera et minor semicirculo. Proportio ergo sinus arcus BF ad sinum arcus EF producitur ex proportione sinus arcus AD ad sinum arcus AE et proportione sinus arcus BC ad sinum arcus CD. Ergo proportio sinus arcus BF ad sinum arcus FE est tanquam proportio sinus arcus AD ad sinum arcus CD. Si ergo sinus arcus CD ducatur in sinum arcus BF et productum dividatur per sinum arcus AD, exibit sinus arcus EF, qui arcus est elevatio arcus CD. Quelibet enim portio zodiaci sive cuiuspiam alterius circuli declivis et sua elevatio ab eisdem circulis meridianis intercluduntur. Quoniam ergo circuli meridiani sunt

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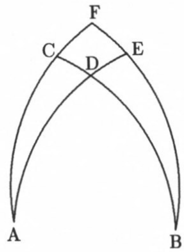
3 Ad scientiam extrahendi ...] tractatus de ortu signorum *marg.* **E** (*man. rec.*)
6 numeri] trium **MSS** *11* residuum] residui **E**

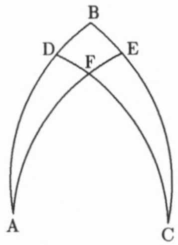
in astrolabio in potentia linearum rectarum, per centrum equatoris transeuntium et puncta cuiuslibet circuli magni vel actu vel potentialiter opposita pertingentium, facile erit omnem circulum declivem per gradus suos in plano distinguere, si eorum declinationes fuerint note.



[Fig. 10]

II Similiter si volueris dividere circulum orizontis secundum elevationes orbis recti ad latitudinem notam, pones complementum latitudinis note quemadmodum declinationem <totam>, et exibit cum eo declinatio singulorum graduum. Facies ergo cum hac declinatione elevationes signorum, ut fecisti ante hoc, et erit divisio orizontis ut divisio orbis signorum. Ratio: operatio elevationum apparet de figura sectoris.

Demonstratio. Scita maxima declinatione orizontis sive alterius circuli declivis declinationes omnium graduum eius note erunt. Sit enim A polus equatoris, sitque AB quadrans circuli meridiani, CB quadrans equatoris, CD quadrans circuli declivis; sit ergo DB maxima declinatio. Protrahatur itaque arcus AE vice unius circuli meridiani secans quomodolibet arcum CD ad punctum F. Igitur proportio sinus arcus AB ad sinum arcus BD producitur ex proportione sinus arcus CF ad sinum arcus CD et proportione sinus arcus AE ad sinum arcus FE. Ergo proportio est sinus arcus AB ad sinum arcus BD tanquam proportio sinus arcus CF ad sinum arcus FE. Data ergo qualibet proportione circuli declivis facile deprehenditur per primum capitulum quantum de equatore cum ea a duobus circulis meridianis includatur. Possumus ergo orizontem latitudinis note per suos gradus in plano distinguere.



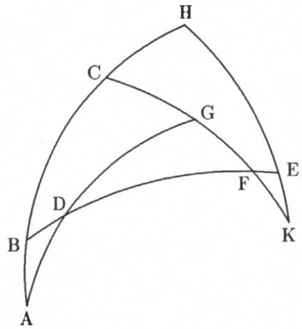
[Fig. 11]

III Et scientia elongationis stellarum a linea directionis: est ut accipias numerum graduum a principio capricorni ad gradum stelle per gradus equales et queras simile eius in elevationibus orbis recti et arcuabis illud ad gradus equales, et quod provenerit tibi vocabis gradum stelle equatum. Et accipies declinationem eius. Tunc si fuerit declinatio et latitudo in parte una, colliges utramque; et si diverse, minues minorem de maiore. Quod ergo remanserit vocabis comprehensum, et scias quod in parte maioris est semper. Et minues declinationem totam de xc et pone residuum sinum et voca primum; et pone comprehensum sinum et voca secundum; et minue declinationem gradus stelle equati de xc et pone reliquum sinum quem vocabis tertium. Duc ergo primum in secundum et divide per tertium, et quod provenerit arcuabis, et erit arcus longitudinis stelle a linea equalitatis in parte comprehensi.

Demonstratio. Sit A polus australis, B polus zodiaci, D stella, E gradus stelle equalis, nam KEH est quadrans zodiaci. Queritur itaque DG, nam arcus KGC est linea directionis, id est equator. Est itaque arcus FE equalis declinationi gradus stelle equati; que dematur a latitudine stelle DE: remanet comprehensum DF. Queritur itaque DG. Quoniam ergo duo arcus AG et FB secant se ad punctum D inter duos arcus AC et FC et quilibet istorum iiii$^{\text{or}}$ arcuum est portio circuli magni semicirculo minor, quatuor isti ar-

65 residuum] residui **E** *66* comprehensum] comprehensi **MSS** *67* equati] equalis **MSS**

56 stellarum] **Ar.** *sing.* *56* linea directionis] *khaṭṭ al-istiwā’* **Ar.**; *cf.* linea equalitatis *infra* cus constituunt sectorem. Ergo proportio sinus AC ad sinum BC producitur ex proportione sinus arcus AG ad sinum arcus DG et proportione sinus arcus FD ad sinum arcus BF. Est autem arcus AC equalis arcui AG: ergo proportio sinus arcus BC ad sinum arcus GD est tanquam proportio sinus arcus BF ad sinum arcus FD. Est autem arcus FB complementum arcus FE: si ergo sinus BC ducatur in sinum comprehensi FD et productum dividatur per sinum complementi arcus FE, exibit sinus arcus GD qui querebatur.



[Fig. 12]

Sit item EG latitudo stelle sitque E stella et arcus GF sit tanquam declinatio gradus stelle equati; erit ergo totus arcus EF comprehensum. Queritur itaque arcus EH. Quoniam ergo duo arcus AF et HB secant se ad punctum E inter arcus AD et HD, habemus sectorem. Igitur proportio sinus arcus AD ad sinum arcus BD producitur ex proportione sinus arcus AF ad sinum arcus FE et proportione sinus arcus HE ad sinum arcus HB. Quoniam ergo arcus BD est equalis arcui HB, erit proportio sinus arcus AD ad sinum arcus HE tanquam proportio sinus arcus AF ad sinum arcus EF. Est autem sinus arcus AD equalis sinui arcus BC, idem enim sinus erit totius arcus si addas alicui arcui xc et residui arcus si illum aliquem arcum tollas de xc; simili ratione idem est sinus arcus AF et sinus arcus residui si arcus FG tollatur a xc: si ergo sinus arcus BC ducatur in sinum arcus FE et productum dividatur per sinum complementi arcus FG, exibit sinus arcus EH quem querimus.

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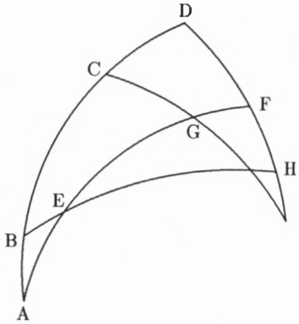
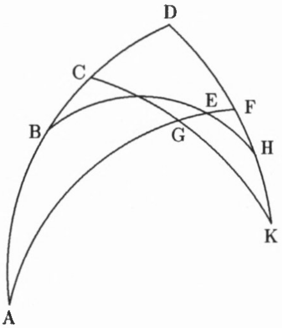
96 BC] *BD* MSS



[Fig. 13]

Sit item stella inter quadrantem equatoris KD et quadrantem zodiaci KC, sitque locus stelle E; erit ergo comprehensum EF. Igitur proportio sinus arcus AD ad sinum arcus BD producitur ex proportione sinus arcus AF ad sinum arcus EF et proportione sinus arcus HE ad sinum arcus HB, et cetera prout demonstratum est prius.



[Fig. 14]

IIII Et scientia gradus cum quo mediat stella celum: est ut sumas quod est inter gradum stelle equatum quem prediximus et finem geminorum aut sagittarii, utri fuerit propinquior ut sit minus quam xc, et pones ipsum sinum et voca primum. Et

108 equatum] equalem **E**

pones longitudinem a linea equalitatis sinum et voca secundum. Et minues longitudinem stelle a linea equalitatis de xc et pones residuum sinum et voca tertium. Et duc primum in secundum et divide per tertium, et eius quod exierit accipies tres octavas et medietatem octave, et quod exierit arcua et serva. Postea vide gradum stelle equatum cuius meminimus, qui si fuerit a primo capricorni ad finem geminorum et fuerit eius elongatio ab linea equalitatis septentrionalis, minues arcum quem servasti de gradu stelle equato. Et si fuerit longitudo meridiana, addes super illum. Et si fuerit stella a primo cancri ad finem sagittarii, fiet econtrario in additione et diminutione, et quod provenerit est longitudo stelle a principio cancri vel capricorni cum elevationibus circuli recti. Convertes ergo illud ad elevationes graduum equalium, et gradus exiens est gradus cum quo mediat stella celum.

Demonstratio. Sit A polus zodiaci, B polus australis, C finis sagittarii; sitque CD maxima declinatio, nam LD quadrans est equatoris et LC quadrans zodiaci; sitque E locus stelle, et H gradus stelle equalis. Erit itaque arcus EF elongatio stelle ab equatore. Queritur autem punctus K, id est gradus cum quo stella mediat celum, qui notus esset si sciretur punctus F. Amplius arcus GD est equalis arcui qui est inter gradum stelle equatum et finem sagittarii; est ergo notus, nam gradus stelle equalis ponitur esse notus. Oportet ergo queri arcum FG per sectorem sic: proportio sinus arcus FG ad sinum arcus GD producitur ex proportione sinus arcus FE ad sinum arcus EB et proportione sinus arcus AB ad sinum arcus AD. Ducatur ergo sinus arcus GD in sinum arcus FE et productum dividatur per sinum arcus EB, et quod exierit vocetur M; est ergo proportio sinus arcus FG ad M tanquam sinus arcus AB ad sinum arcus AD, qui est equalis sinui arcus BC. Est autem proportio sinus arcus AB ad sinum arcus BC tanquam vii ad xvi: ergo sinus arcus FG est tres octave et dimidia octave totius M. Ergo arcus FG notus est; ergo arcus FD notus. Quem si arcuaveris ad gradus equales, proveniet arcus KC; ergo punctus K notus est quem querebamus. Reliquas variationes lectori querendas relinquimus.

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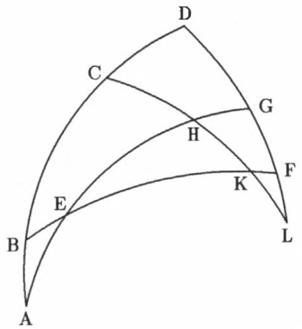
112 longitudinem] longitudines MSS



[Fig. 15]

V Et scientia gradus cum quo oritur stella: est ut minuas medietatem <arcus> diei stelle de eo quod est apud gradum mediationis graduum elevationum circuli recti. Et si non fuerit unum, addes ccclx, et quod remanserit queres eius simile in tabula elevationum tue regionis in gradibus elevationum et arcuabis ad gradus equales, et quod exierit de gradibus equalibus, in eo gradu ascendet stella. Et scientia cum quo gradu occidet stella: est ut addas medietatem arcus diei supra id quod est apud gradum mediationis elevationum circuli directi, et quod collectum fuerit arcuabis in tabula elevationum tue regionis, et quod provenerit est nadair gradus occasus. Intelligas. Explicit.

b) Notes to the Arabic text

Annotations to the Arabic text as edited by Vernet and Catalá, pp. 26–28 (according to page and line numbers).

26,7 تحفظه : وتحفظه MS 9 هى : فهى MS 13 البروج : الربع MS 15
حتى : *del.* MS, *supra* ثم 16 ذلك : بذلك MS 17 قسمه : قسمة 19
الكوكب : من اول الكوكب *del.* MS 21 قسمه : قسمّه

27,8 اواخر : أو آخر 12 يضرب : تضرب; فما اخرج : *leg.* فما خرج (MS *del.* ا
ante خرج); احذت : اخذت 13 ونصف : ونصف ثمنه MS 14 ...الجوزاء :
ذلك : دلك 17 هذه القوس : نقصت هذه القوس MS 15 الجوزاء وكان MS
21 تطلع : يطلع

28,3 فما : فمع MS; تطلع لكوكب : يطلع الكوكب MS 9 تاخد : تاخذ 12
يكون : تكون 13 هذا : بهذا MS 14 تقريبا : بتقريب MS

Table on p. 44: at 30°, 10″: 9 MS; at 55°, 29″: 21 MS; at 65°, 33″: 38 MS; at 75°, 13″: 58 MS; at 90°, 5″: 0 MS

Comments

1. The diagrams

The following remarks are intended to serve as an *apparatus* for the diagrams, which have been reproduced in the edition[1] derived from the diagrams in **Ar.**, **A** and **E**. Relative sizes of constituent lines and actual sizes of the diagrams in the manuscripts are not normally mentioned. Here, as elsewhere, the transliteration of **ABNP** is used for the diagram letters[2]. A diagram letter in inverted commas refers to the letter written on the diagram, and without inverted commas to the point so defined.

Fig. 1 is true to the Arabic (f. 71r). It is Ptolemy's diagram (Heiberg, p. 250) for §14[3], but with HT cutting the smaller circle and not falling outside and with the whole rotated anticlockwise through a right angle. **A** and **E** have no separate diagram for this (first) part of Maslama's note; the diagram

[1]They are added to the Latin versions; in the corresponding places in the Arabic text and the English translation there are only references to the respective Figures.

[2]The complete set of letters and their correspondence in Arabic and Latin is as follows: 17 letters are the same as in the text of the *Planisphaerium*, *alif* $= A$, *b* $= B$, *j* $= G$, *d* $= D$, *h* $= E$, *z* $= Z$, *ḥ* $= H$, *ṭ* $= T$, *k* $= K$, *l* $= L$, *m* $= M$, *n* $= N$, *s* $= C$, *ʿayn* $= Y$, *f* $= F$, *q* $= Q$ and *r* $= R$. In some notes Maslama uses four more letters: *w* $= O$, *ṣ* $= X$, *t* $= P$ and *th* $= S$. Cf. also above, p. 9.

[3]Numbers following § indicate the sections into which Heiberg divides the text.

for §14 of the *Planisphaerium* is rotated anticlockwise through a right angle in **A** (158vb) and reflected left-to-right in **E** (93r).

Fig. 2 follows **A** (160vb) and **E** (93r). In **Ar.** (71v) HZ is drawn as the continuation of TH.

In Fig. 3 Maslama took over Ptolemy's diagram and in his elaboration used the letters "Z", "T" and "K" again, in the upper part of the figure. In **Ar.** (72v) these appear in their original positions (marked in our diagram by "Z'", "T'", "K'") as well as in their new positions. In **A** (160va) and **E** (94r) they occur only once, in their new positions. In **A**, "E" and "B" are missing; "C" is displaced and is near M; and $K'H$ and $T'K'$ are drawn parallel to AG.

Fig. 4 appears in all classes of manuscripts, no doubt because Maslama's note 9 does too. Heiberg's diagram (p. 256) is a simplification, to show only those points and lines used in Ptolemy's text. In **Ar.** (73v) "P" is outside the outer circle; "O" is caught in the binding of the manuscript; "B" is missing in its proper place; "B" is written for "F". In **A** (159va) triangles FKM and ERY are drawn to the left of DB, **E** (94r) has two diagrams, in both of which the circle through OYC is drawn. The first is like our Fig. 3, but the second has L between B and G and triangles FKM and ERY on the left of DB, the whole being rotated clockwise through a right angle so that DL represents a different position for the zodiac-parallel; P lies on circle QYN. In **D** (9v) there is a diagram essentially similar to the second in **E**. In all manuscripts EY and TL are drawn complete.

In **Ar.** (74v) Fig. 5 is drawn without "B" in its proper place; "B" is written for "F"; "A" is missing. In **A** (161ra) "Z" is misplaced and labels the intersection of HK and circle DG. In **E** (94v) the diagram is drawn as if rotated clockwise through a right angle and then reflected left-to-right.

Fig. 6, in which we have drawn YE with a break, follows **E** (94v). In **Ar.** (75v) "D" is misplaced upwards and labels the intersection of the small semicircle with line EF. In **A** (161ra) the diagram is drawn rotated anticlockwise through a right angle.

Fig. 7 represents the diagram in **Ar.** (76v). In **E** (55v) points E and Z and letters "E" and "Z" are missing. In **A** (161va) the whole is rotated through two right angles.

Fig. 8 is as in **Ar.** (77r). **E** (55v) has two diagrams, of which the second is as drawn here; in the first "K" and "G" are missing. In **A** (161va) the diagram is drawn rotated anticlockwise through a right angle.

In Fig. 9 "A", which appears in **Ar.** (78v), has been retained, although the text does not refer to it and it is missing in **A** (162ra) and **E** (56r). In **A** there are no axes, the figure is rotated anticlockwise through a right angle and the curvature of the two arcs through Z is in the opposite sense. In **E** there is no horizontal axis and "B" is displaced to label the upper intersection of the two complete circles.

Figs. 10-15 are copied from **E** without essential change.

2. Maslama's notes

In the following a brief summary of the content of the notes is given together with an indication of the relation of Latin to Arabic[4]. Notes 1, 2, 3, 6, 7, 9, 11 appear in all manuscripts, worked into Hermann's translation at the end of the propositions concerned[5]; 4, 5, 8, 10, in a different translation, are found in the margins of **EF**, which also carry substantial parts of notes 7 and 9 and fragments of others (for the alternative translations in **EF** of the *Planisphaerium* itself, see Appendix I); 5, 8, 10, in yet another translation, are collected at the end

[4]Notes 2, 3, 5, 8, 9 and 11 were briefly described by Anagnostakis, pp. 139-144.

[5]These notes are in Commandinus' edition and therefore appear also in Sinisgalli-Vastola's translation.

of the text or elsewhere in **ABNP**, which also (except **P**) have another translation of part of note 9 *in margine*. All the notes or fragments peculiar to **ABNP** are in the same style, which is very literal; the **EF** notes are also consistent, but the style is less literal; the "internal" notes have the same characteristics as Hermann's translation of the *Planisphaerium* itself — sometimes literal, more often a loose paraphrase, and sometimes more an interpretation than a translation[6].

Note 1 explains Ptolemy's sentence (229:21–25[7]) about finding the beginnings of the signs on the plane by drawing circles parallel to the equator at the appropriate distance from it; cf. the first method of dividing the ecliptic mentioned by Maslama in the Extra-Chapter. The note does not make the matter much clearer. The Latin is a summary, not a translation. It omits the last sentence about the practical difficulty of finding where mutually tangent circles meet. This sentence seems irrelevant, for the only occasion when this difficulty would arise is in finding the first points of Cancer and Capricorn, which are found by intersection with the axis of the map.

Note 2 is an intended clarification of the proposition of §3, that on the plane a horizon circle intersects the zodiac at points that represent opposite points on the sphere. Ptolemy's proof runs as follows. Let T be the intersection of horizon HAG and line HE, where H is one of the two intersections of the zodiac BHD with the horizon, and E the centre of the equator $ABGD$ (see Fig. i, with M for T). Then

$$HE \cdot ET = AE \cdot EG = BE \cdot ED.$$

Therefore T lies on the zodiac — a conclusion apparently relying on the converse (not proved by Euclid) of *Elements* III 35.

[6]For the correspondence of the text of the *Planisphaerium* and Maslama's notes, see the table on p. 97. In the sources Maslama's notes are unnumbered; the numbers have been added by the present editors.

[7]Such numbers indicate the page and line number in Heiberg's edition of the Latin *Planisphaerium*.

This reduces the case considered in §3 to that of §2. The text is not entirely clear about the definition of T and Maslama defines the intersection of HE with horizon and zodiac as M and Z respectively. Then

$$ME \cdot EH = AE \cdot EG = BE \cdot ED,$$
$$\text{but } ZE \cdot EH = BE \cdot ED,$$

and so $EM = EZ$. Hence HE produced meets the horizon only at T, which Maslama evidently takes to be the intersection of HE and the zodiac. Latin and Arabic in general are

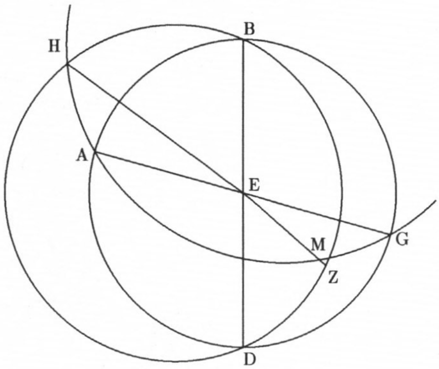


Fig. *i*

close, but there are differences: the Latin inverts one mathematical line, omits another, and adds a sentence at the end summarizing the proposition.

Note 3 is placed, according to its introduction in Arabic, after the fourth *shakl*; it correspondingly appears in Hermann's translation after §7. Maslama applies the method of §4, for finding the end-points of the diameters of the representations of parallel-circles on the plane, to the case in which the declination of the parallel-circle is 89°. From the case of declination 90°, which he evidently considers as the limiting case, he shows what Ptolemy stated at the beginning of the book (227:26), that the south pole cannot be represented on the plane. The Latin follows the Arabic except towards the end: *Si enim linee recte* ... does not correspond exactly with Arabic lines 35–41.

Note 4 is explicitly on §8, which is about the right ascension of parts of the zodiac. After showing the symmetries of the ascensions of the four quadrants, Ptolemy shows how to calculate the ascension of a specified point of one quadrant by plane trigonometry and then gives an "easier" method, also by plane trigonometry and arithmetically not very different from the first. In Maslama's note perhaps the "easier way" refers to this second method or perhaps to a supposed simplification of it by the calculation of §4. The "previous question" probably refers to §4 (or §4–7), in which the diameters of the equator-parallels are determined; and the "second question of his book" probably refers to §2, which comprises the justification of one type of symmetry[8]. The Latin exists only in the **EF** translation.

Note 5. §14 solves the problem: given the greatest circle in the plane and the distance (DZ) of its correlate on the sphere from the south pole, to find the radius (EZ) of the equator. Ptolemy's construction may be summarized (see Fig. 1, above): $GH \parallel ED$; $HT \perp ED$; $EL = TK$. To show that EL is the required radius, it is proved that arc MN is similar to arc DZ, so that circle $ABGD$ conforms to the conditions of a projection of a circle through N projected from M. In the course of the proof GM is shown to be parallel to DZ; from this follows $\angle GME = \angle ZDE$, and so arc BGZ is similar to arc CLN. Maslama's construction is much simpler: $GM \parallel DZ$, and EM is the radius of the equator. The rest is as in Ptolemy. Maslama then gives a second construction, which is simpler still: if circle $ABGD$ and arc GZ (see Fig. 2) are as in Fig. 1, draw BZ, and EH will be the radius of the equator. Arc KT is similar to arc GZ, since $\angle KHT = \angle GBZ$.

There are two Latin translations, in **EF** and in **ABNP**. Of these the latter adds a "Q.E.D." formula after the first

[8]Thus *mas'ala* here appears to mean the same as *shakl*.

construction and proof. **EF** omits the last line of the first proof and virtually the whole of the second.

Note 6. In §15 Ptolemy finds the projection of the pole of the zodiac and points out that circles through this point and through opposite points of the zodiac will cut the zodiac orthogonally. Maslama makes the obvious remark that such a circle will pass through a star if, and only if, it passes through the degree of the star (i.e. the point on the zodiac with the same longitude). He adds that a meridian passes through a star if, and only if, it passes through the co-culminating point of the zodiac. The Latin is a fairly free translation, but complete.

Note 7 is unambiguously attached in the Arabic to §16; further, it refers to the pole of the ecliptic "which is point K in the question [*mas'ala*] before this" and this reference must be to §15 of the *Planisphaerium*. §16 is about the projection of a zodiac-parallel and its intersection with the equator-parallel that it bisects. The note explains how to place a star on the plane map by its longitude (see previous note) and latitude (defined by a zodiac-parallel). **EF** adds in the margins two fragments of its own characteristic translation.

Note 8 is on §17, in which Ptolemy proves that the centre of one zodiac-parallel ($Z'H$ in Fig. 3) cannot be the centre of another ($T'K'$). To this end he proves the inequality

$$(NC+CM)\cdot MC > (NC+NL)\cdot NL,$$

from which, he says, it follows that $CM > NL$ — a correct, but not immediate, deduction. Maslama gives an alternative proof of the proposition. Where Ptolemy assumes $DM > DL$, Maslama assumes both this and $DN < DC$. His procedure is to construct $\triangle DTZ$ as a copy of $\triangle DNL$ and to drop perpendicular TO. Now since $DC > ND$, $\angle DNC > \angle DCN$ [*Elem.* I 18]; and so $\angle DCM > \angle DNL = \angle DTZ$. So when TK is drawn parallel to MC, its other end falls below Z [i.e. $DK > DZ$]. Also

$CD : TD = MC : TK$, and so $CM > TK$.
But $TK > TZ$ [since it lies further from the perpendicular TO].
Therefore $MC > TZ$, which $= NL$.

Further, it can be proved that $KT > ZT$ without drawing the perpendicular TO, since $\angle DZT$, which $= \angle DLN$, is acute [and $DK > DZ$]. Finally, Maslama adds the corollary that lines defined as CM is are shorter when they are nearer the centre.

The Latin translation in **ABNP** is mostly literal except for the opening words of the alternative proof of $KT > ZT$. Both translations say that the perpendicular TO is situated like the perpendicular NY, although the Arabic has it, unreasonably, the other way round.

Note 9 is evidently assigned to §18, as is shown by the quoted opening words. The quoted closing words match neither those of the available Arabic nor those of Hermann's translation. They are closest to 256:9–10 in Hermann, a passage in which there is not an exact correspondence of Latin and Arabic. §18 is about a zodiac-parallel that intersects the greatest always-invisible parallel to the equator[9], i.e. the parallel that touches the horizon from below. In Fig. 4 semicircle ZMH is half the greatest always-invisible circle and NYQ is its image in the plane of the equator. The zodiac-parallel is the circle standing on diameter TL, so that KM represents its common section with the greatest always-invisible circle. According to the passage just referred to (256:9–10; cf. **Y** 95v,1–3), not only does the representation of the zodiac-parallel pass through the image C of one end (L) of the zodiac-parallel on the sphere, but the arcs it cuts off the greatest always-invisible circle are similar to those cut off in the sphere. Ptolemy constructs a point Y on the image of the greatest always-invisible

[9]§18 has suffered in transmission. It is not absolutely certain that the *greatest* always-invisible circle is meant.

circle so that NY is similar to HM (the constructions are different in Arabic and Latin). Maslama proves that Y lies on the circle of diameter CO, though he states his conclusion in the weak form that points C, Y, O are on the circumference of a circle. Since the theorem is valid for any parallel to the equator, it is tantamount to a proof of the property of stereographic projection that circles map into circles, but it is not presented as such[10]. Maslama's proof, which is entirely in keeping with the style of the *Planisphaerium* — cf. the argument in §2 sketched above and the proof of §16 —, may be summarized as follows[11]:

> Let $TX \parallel AG$, so that $\overset{\frown}{DX} = \overset{\frown}{DT}$.
> Since $\angle DLT = \angle DTX$ [III 27] and $\angle DTX = \angle DPH$ [I 29], therefore $L\ S\ T\ P$ lie on a circle [III 21 *conv.*].
> Therefore $PK \cdot KS = KT \cdot KL = KZ \cdot KH$ [III 35].
> Therefore $OR \cdot RC = RQ \cdot RN$, since PH and ON are similarly divided.
> But $\triangle REY \sim \triangle KFM$, since $\angle F = \angle E$ (Ptolemy's construction[12]) and the sides about these angles are proportional, since $RE : FK = EQ : FZ$.
> Therefore $\angle YRE$ is right.
> Therefore $QR \cdot RN = RY^2$.
> Therefore $RO \cdot RC = RY^2$.
> And since $RY \perp [OC]$, C, Y, O lie on a circle [of diameter CO].

We note that lines 109–110 repeat material from the *Planisphaerium* (in the recension followed by Hermann), albeit not exactly, *viz.* the construction of point O and the statement that the circle through C and Y goes through O. By "the question before this one", which Maslama cites in this connection, §16 must be meant, although it is not immediately

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[10]See Lorch.
[11]The references to Euclid in square brackets are not in the text.
[12]I.e. in the recension followed by Hermann and Maslama.

before §18. For Maslama's procedure here is very similar to Ptolemy's or his own in §16 (see the comment on note 10) and he explicitly says that the conditions in this proposition must be like those of "the foregoing [question]". The similarity of conditions and proof is no coincidence, for §16 is mathematically a special case of §18.

Note 9 was translated as a note within the text, with the omission of lines 108 (the beginning) to 113. Also omitted is the line "since $RE : FK = EQ : FZ$". At the end there is an extra paragraph (*Ex his manifestum* ...), which is discussed below in the comment to note 10. In the margins of **ABN** and **EF** there are further translations, in their usual styles, of lines 108–117 and 108–114 respectively (= notes 9^1 and 9^2 in the above edition).

Note 10 has no introductory passage in Arabic to identify the text commented upon, as all the other notes have, but is linked, as an addition to a preceding note, by the formula "Maslama also said". Maslama begins the note "In order to complete what we must complete in this question ...". There follows the statement of the result about to be proved:

> that is that we show that this circle parallel to the zodiac will also [*sic*] bisect the circle parallel to the equator circle which it bisects in the solid sphere.

The reference to completion is similar to that at the beginning of note 9. In fact note 10 states the proposition for §16 and proves it for the case of a southern zodiac-parallel. Ptolemy had proved it, with a similar, but slightly less elegant, proof, for a northern parallel. The essence of Maslama's proof is (see Fig. 5):

$\angle DLT = \angle DTM$ [III 27].
$\angle DTM = \angle DNK$ [I 29].
Therefore L, Z, T, N are concyclic [III 21 *conv.*].

Therefore $KN \cdot KZ = KT \cdot KL = KH^2$ [III 35].
Therefore $OE \cdot EC = QE^2$, since NZ and OC are similarly divided, and so $= EF^2$.
Therefore the circle through C, Y, O [on diameter CO] passes through F.

Point Y is not defined, but if it is understood to be somewhere on the circle of diameter CO, this does not detract from the proof. In §16 the definition of the equivalent point is also vague.

The translation in **ABNP** omits a concluding line of the argument and also the postscript comparing the southern zodiac-parallels with the almucantars of a southern astrolabe. The last sentence of the Arabic, which is not in either Latin translation, is almost certainly extraneous.

The note appears to be displaced in Arabic. Probably the **EF** translator also saw it after note 9, for **E** has it in the upper margin of the page (f. 94v) on which the text begins in the middle of note 9, and not on the page (f. 93v) on which the entire §16 is written. Similarly, the note appears in **F** on f. 219r, on which note 9 is written, and not on f. 218v with §16. Of the **ABNP** group **NP** insert note 10 in the text in the middle of note 9, after the translation from the Arabic noted above and before *Ex his manifestum* ... . This perhaps means less than at first appears, since **AB** have this note, together with notes 5 and 8, at the end of the text of the *Planisphaerium*. It is probable that Hermann also had an Arabic text before him with note 10 coming after note 9. For after *Ex his manifestum* ... we find

> that in the sphere when the parallel to the equator and the parallel to the zodiac are about the same centre, they cut each other in half,

which is of course nonsense (unless the equator and zodiac themselves are being considered), but seems to be a garbled version of the enunciation of note 10 given above. Hermann then says that such a thing is not possible on the plane and that he therefore will not bother with Maslama's proof, *quam ... hic interponit*. What Hermann did not want to pursue was certainly something of Maslama's which he presumably misunderstood. Interestingly, the Hebrew translation puts note 10 between §16 and note 7[13]. But this translation is probably not an independent witness to the Arabic text, for it has elements clearly dependent on the Latin version[14]. So the placing of note 10 in this version may have been the work of the translator.

But it is still possible that note 10 was originally placed after note 9, where both Arabic and Latin put it. This hypothesis accords well with its enunciation, "we prove that ... *this* circle ... will *also* bisect the circle parallel to the equator ...", with the strictly analogous reasoning in the two notes, and with the lettering of the diagram of note 10, which has more in common with that of note 9 than that of §16. The curious position of note 10 could — just — be explained by supposing that the author thought it the only remaining result in the general subject (*mas'ala*) under discussion that needed to be proved. But it remains odd that the first-class mathematical material in the two notes should be introduced so casually. Further, it will be remembered that notes 5, 8 and 10, which with note 9 are of a markedly higher quality than the rest, received different treatment at the hands of the Latin

[13]E.g. in MS London, BL add. 26984 (no. 1002 in Margoliouth's catalogue of 1915) note 10 occurs on ff. 17r,23–17v,13 and note 7 begins on f. 17v,17, the intervening lines being occupied by an unidentified sentence; cf. MS Bodleian, Opp. add. 4[to] 175 (no. 2582 in Neubauer's catalogue), ff. 8v–9r.

[14]We refer to the section corresponding to note 9[3], lines 28ff., where "the Christian translator" (*ha-ma'tīq ha-noṣrī*) is explicitly mentioned; cf. Neubauer, col. 1124 (on MS Oxford, f. 10r) and the corresponding place in MS London, f. 19v.

translators and presumably appeared differently in the Arabic manuscripts used by them. Even note 9, or notes 9 and 10, was not treated by Hermann exactly like the other notes he included. For he breaks off, saying he wants to keep to Ptolemy[15]. We may therefore speculate whether all the eleven notes going under Maslama's name were really written by him or whether two authors of different competence were involved.

Note 11 is assigned to §19. In this section Ptolemy proves that a zodiac-parallel through the south pole is projected into a straight line and gives a construction to find it. Maslama adds that the arcs cut by the straight line from the image of the always-hidden circle (as he assumes the equator-parallel in Ptolemy's diagram to be) are similar to those cut from the always-hidden circle on the sphere by the zodiac-parallel. No doubt he wished to complete the result of §18 and note 9 by showing that the similarity of arcs is also true for the special case of the zodiac-parallel through the south pole. Maslama's proof may be summarized (see Fig. 6):

Since $ZH \parallel NE$, $NE : CE = HF : FK$.
But $NE = EY$ and $HF = FM$.
Therefore $YE : EC = MF : FK$.
And $\angle YCE = \text{rt.} = \angle MKF$.
Therefore $\triangle MFK \sim \triangle YEC$ [VI 7].
Therefore $\angle YEC = \angle MFK$.
Therefore $\overparen{NY} \sim \overparen{HM}$ and so also $\overparen{YQ} \sim \overparen{MZ}$.

As for the extra sentence in Latin, it is hard to suggest an emendation to make sense of it.

3. Maslama's Extra-Chapter

Maslama begins by remarking that Ptolemy had mentioned two ways of graduating the zodiac and its parallels, but had not done the same for the horizon. Since the two

[15] He also leaves out the introductory material in note 9, but he does this elsewhere, e.g. in note 2.

are strictly analogous, this must mean that Ptolemy had not actually performed the calculation or construction in the latter case. The first way of dividing the zodiac in the plane may be expressed in modern terms: to find the point x degrees from the equinox, intersect the representation of the zodiac with the representation of the equator-parallel of declination $\delta(x)$ from the equator[16]. We notice that the method of determining the radii of the representations of the equator-parallels was given in §4 of the *Planisphaerium*, with examples in §5–7, and that the determination of points on the zodiac by declination was used in §8–9 in the determination of right ascensions. The second way is to draw through the pole a line that cuts off arcs from the equator equal to $\alpha(x)$. As Maslama says, this is the more accurate method, since the exact point of intersection of zodiac and equator-parallel may not be easy to find. Now Maslama gives a third way[17]: he constructs a great circle cutting arcs from equator and zodiac that are equal on the sphere, so that the graduation of the one can be copied onto the other. In Fig. 7 circle ABG is the equator, circle ADG is the zodiac, B and D are the midpoints of ABG and ADG, and H is the midpoint of BD. A great circle through T, the pole of AHG, will cut off equal arcs (AM and AK) from equator and zodiac. T is midway between E and Z. The image of such a circle is constructed — see Fig. 8, in which circles ABG and ADB are the equator and the zodiac respectively — by first finding the image of T: M is taken midway between A and Z, the image of the pole of the zodiac; the intersection of BT with the common axis $GDEK$ is T. Now L and N are taken so that $AL = BN =$ the arc to be copied onto the zodiac; and a circular arc is drawn through L, T, N: its intersection with circle ADB yields the desired arcs AY, BF (Y and F, of course, represent opposite points on the sphere). Later, the

[16] δ and α represent the declination and ascension of a point.

[17] See Samsó for a clear exposition of this method.

method is found both in the *Libros del Saber* and in the *De plana spera* of Jordanus, both of the mid-thirteenth century[18].

After the curious remark that the division of the horizon is needed "to know the Sun", Maslama repeats the three ways for the horizon, taking the complement of the geographical latitude instead of the obliquity of the ecliptic. In explaining this Maslama gives the value 24°. We may notice that Ptolemy uses this value in §20 of the *Planisphaerium*, but other values elsewhere, e.g. in §4[19]. At the end of the third way Maslama says that an arc should be drawn through the two points just found on the horizon and through the horizon's pole: this will cut the horizon and its parallels through their respective degrees.

The last section is on three methods of finding the position of a star on the rete. In the first the star is found at the intersection of two circles: the appropriate zodiac-parallel and the circle through the star's degree, the opposite point and the pole of the zodiac (cf. Maslama's note 7 and also *Planisphaerium* §15 and Maslama's note 6). The second is the usual method with declination and co-culminating point of the zodiac[20]. For the third method the co-ascending and the co-descending points of the ecliptic must be known. The co-ascending point is put on the eastern horizon and an arc of the horizon is drawn on the rete; then the co-descending point is put on the western horizon and again an arc of the horizon is drawn: where the two arcs meet is the position of the star. It may well be asked what kind of rete Maslama has in mind for this method. No doubt we may assume that it was made of paper or some similar material before the markings were transferred to metal (if indeed the usual cut-away metal rete was the ultimate purpose here). To draw each of the two arcs

[18]See Samsó; Thomson, pp. 124–128, 143–144.
[19]See Anagnostakis, p. 199; Heiberg, p. 234.
[20]See, e.g., al-Farghānī's *Al-Kāmil*, MS Berlin Ldbg. 58, f. 46v.

of the horizon on the paper rete, the constructor would presumably rotate over it some kind of template containing the stereographic projection of the horizon; and then he would either use the marked horizon or, possibly, the marked centre of the horizon for the actual drawing of the arc. Alternatively, the method may have been conceived as a purely mathematical exercise, without reference to any practical application[21]. We may note in passing that two horizons appear on one diagram in §10 of the *Planisphaerium*.

4. Maslama's Astrolabe Chapters

In this short text Maslama applies the sector figure (*al-shakl al-qaṭṭāʿ*), i.e. Menelaus' theorem, to the determination of various arcs required in laying out a plate generated by stereographic projection. The use of the arcs for this purpose is explained in the Extra-Chapter. In the Arabic text of the Astrolabe Chapters only rules are given — though it is once stated that a rule follows from the sector figure —, but in the Latin version proofs are added by an unknown author for the first four chapters. From the diagram letters (A, B, C, D, E, F, ...) and from the general style this commentary appears to be of Latin origin. In the following R is the radius of the sphere, ε is the obliquity of the ecliptic, β means latitude with respect to the ecliptic; the medieval sine function is represented by "*Sin*", so that $\mathit{Sin}\,x = R\sin x$, and "*Cos*" means the sine of the complement. Menelaus' theorem for Fig. 10,

$$\mathit{Sin}\,BF : \mathit{Sin}\,EF = (\mathit{Sin}\,AD : \mathit{Sin}\,AE) \cdot (\mathit{Sin}\,BC : \mathit{Sin}\,CD),$$

will be represented

$$\mathit{Sin}\,(BF : EF = AD : AE \cdot BC : CD)$$

and will be characterized by $FCA/\triangle BED$, which means that line FCA cuts the sides of triangle BED. y sometimes represents the result or intermediate result of a calculation; it does

[21]See also Anagnostakis' remarks, pp. 171–178.

not necessarily stand for a noun in the text, but is used in the modernization of the argument.

In these rules the "ascension" of an arc of the ecliptic is to be understood, as usual, as the co-ascending arc of the equator, but the arcs of both ecliptic and equator are here measured from the solsticial colure and not from one of the equinoxes. This system was used by al-Battānī and others, who drew up tables accordingly[22]. Nallino points out that it is more convenient for some astrological purposes. We may add that the same goes for finding *mediatio* (see below).

I. Right ascension

Rule. To find the right ascension of an arc of the ecliptic of given length [here x] measured from the beginning of Capricorn, first find $Cos\,\delta$, where δ is the declination of the point, and then

$$(Sin\,x \cdot R)\;/\;Cos\,\delta = Sin\,y,$$

where y is the ascension of x. Maslama gives the rule twice, for the examples $x = 30°$ and $x = 60°$.

Justification for the example $x = 30°$. In Fig. 10, A is the south pole, B the beginning of Aries, CD the sign of Capricorn; so $DE = \delta$ and $CF = \varepsilon$. Now

$$Sin\,(BF : EF = AD : AE \cdot BC : CD)\;[FCA/\triangle EBD].$$

$$\text{Therefore } Sin\,BF : Sin\,FE = Sin\,AD : Sin\,CD.$$

$$\text{Therefore } (Sin\,CD \cdot Sin\,BF)\;/\;Sin\,AD = Sin\,EF,$$

and EF is the ascension of CD. The author then outlines the application of ascensions to dividing the zodiac or some other inclined circle (cf. the second *modus* of the Extra-Chapter).

[22]For the use of the beginning of Capricorn in measuring right ascensions of stars, cf. Nallino I, 163–164, with additions in I, p. LXXII, and II, p. XVII. Nallino mentions other astronomers who measured ascension from the colures: Theon, Ibn Yūnus and Kūshyār b. Labbān.

Apart from the use of sines instead of chords, the justification is entirely in the style of the *Almagest*. But we may note that in *Almagest* I 16 Ptolemy finds ascensions by $CDB/\triangle AFE$, which yields the equivalent of

$$Cos\,\varepsilon : Sin\,\varepsilon = (Cos\,\delta : Sin\,\delta) \cdot (Sin\,EB : R).$$

II. Division of the Horizon

Rule. This is done in the same way as the division of the ecliptic, but with the complement of the geographic latitude instead of ε. The method is to find the declinations of the points of division and from these to find the ascension as above (cf. the second *modus* of the Extra-Chapter and the passage added after Chapter V).

In the *justification* the declination of a given point F of the horizon CFD (see Fig. 11) is found, A being the pole of the equator CEB, by

$$Sin\,(AB : BD = CF : CD \cdot AE : FE)\,[BEC/\triangle ADF],$$

etc., just as in *Almagest* I 14. The rest follows as in I.

III. Distance of a star from the equator

The "modified degree" of a star is equal to the arc of the ecliptic whose ascension is equal to the longitude of the star, i.e. its inverse ascension. Although it is measured in *daraj al-sawā'*, a term for degrees of the ecliptic[23], this term refers only to the method of finding the inverse ascension: it is considered an arc of the equator or rather the end-point of such an arc, measured from the solsticial colure. This is shown, for instance, by the implied definition of the direction of its declination (see below) and by Maslama's combining it with other

[23]Cf. al-Battānī in Nallino III, p. 41,6ff., who explicitly says that *daraj al-sawā'* are degrees of the ecliptic. Both al-Battānī and Maslama use *qawwasa* for finding inverse ascensions from tables. In this passage al-Battānī does not use the term *mu'addal* for the modified degree.

arcs of the equator but never with arcs of the ecliptic (see Chapter IV, below). Thus the modified degree of a star may be found on the equator as its intersection with the great circle through the pole of the ecliptic and the star. The declination of the modified degree is the distance of this point along this great circle to the ecliptic. The term for finding the inverse ascension in Chapter IV (*ḥawwilhā ilā maṭla‘ daraj al-sawā’*) is similar to that of al-Battānī (*taḥwīl ilā daraj al-sawā’*)[24].

Rule: if δ is the declination of the modified degree, δ and β are either added or subtracted (see below) to form the "result" (*ḥāṣil*, *comprehensum*); then $Cos\,\varepsilon$ is called the "first", the sine of the "result" the "second", $Cos\,\delta$ the "third"; and

$$(\text{"first"} \cdot \text{"second"}) \;/\; \text{"third"} = Sin\,y,$$

where y is the distance of the star from the equator. The direction of this distance, i.e. whether it is north or south of the equator, is the direction of the "result", which is the same as that of β and δ if they are the same (in which case they are added), or that of the greater of the two if they are different (in which case the lesser is subtracted from the greater). This only works if the direction of δ is taken with respect to the equator; and this in turn implies that the modified degree is considered to be on the equator.

In the *justification* there are three diagrams, to accommodate the various positions of the star. In the first (Fig. 12), A is the south pole of the equator KGC and B is the pole of the ecliptic KEH; D is the star, which is south of both equator and ecliptic and nearer the equator. FE is the declination of the modified degree and $DF = DE - FE = \beta - \delta$, the "result".

$$Sin\,(AC : BC = AG : DG \cdot FD : BF)\ [CGF/\triangle ABD].$$

But $AC = AG$ and FB is the complement of FE. Hence

$$(Sin\,BC \cdot Sin\,FD)/\,Cos\,FE = Sin GD.$$

[24]Nallino, III, p. 41,7.

GD is the distance of the star from the equator. In the second and third diagrams (Figs. 13, 14), A is the pole of the ecliptic CG and B the pole of the equator DFH, though A and B are not specified in the text. Probably they are intended as the respective south poles, so that the star E is again south of both equator and ecliptic but nearer the ecliptic in Fig. 13 and between equator and ecliptic in Fig. 14. $EG = \beta$, $GF = \delta$, EF is the "result", and EH is the required arc. No doubt the other cases, when the star is north of the equator, are considered to follow by the same reasoning on the same diagrams, only with A and B as north poles. The method is that of *Almagest* VIII 5, where only one case is considered.

IV. Mediatio *of a star*

Rule. The sine of the modified degree is called the "first"; and the sine and cosine of the distance of the star from the equator (found in III) are called the "second" and "third", respectively. Then

$$\frac{\text{“first”} \cdot \text{“second”}}{\text{“third”}} \cdot \left(\tfrac{3}{8} + \tfrac{1}{2} \cdot \tfrac{1}{8}\right) = Sin\, y.$$

Then the result y and the modified degree of the star are added or subtracted (see below): the result is the ascension of the *mediatio*. It will be noticed that longitudes and the corresponding ascensions and modified degrees are here measured either from the beginning of Capricorn or from the beginning of Cancer and that Maslama loosely speaks of the modified degree as being on the ecliptic although, as we have seen above, it appears to have been a degree on the equator. Maslama gives rules to determine whether to add or subtract y (FG in Fig. 15) and the modified degree (GD). The question is whether AE and BE, the lines through the star and through the poles of ecliptic and equator respectively, cross before cutting the equator to produce points G and F respectively and whether the arc of the modified degree is measured from the tropic of Cancer or the tropic of Capricorn. Maslama's rule for the southern star between Sagittarius and Aries in Fig. 15 is correct, but in some other cases it seems not to be true.

Justification. A is the pole of the ecliptic quadrant LC, B is the south pole of the equator quadrant LD, and C is the end of Sagittarius; E is the star. It is required to find K, which would be known if F were known. GD is equal to the arc between the modified degree of the star and the end of Sagittarius, and so is known. So

$$Sin\,(FG : GD = FE : EB \cdot AB : AD)\;[GEA/\triangle FDB].$$
$$\text{Now let } (Sin\,GD \cdot Sin\,FE)\;/\;Sin\,EB = M.$$
$$\text{Then } Sin\,FG : M = Sin\,AB : Sin\,AD = \tfrac{7}{16}.$$

Since the constituents of M, called by Maslama "the first", "the second" and "the third", are all known, FG is known; so FD is known; ascension tables will yield K. Until the appearance of M, the method is that of *Almagest* VIII 5. The curious value $\frac{7}{16}$ for $Sin\,AB : Sin\,AD$, i.e. $Sin\,\varepsilon\;/\;Cos\,\varepsilon$, implies a value of 23;37,45.8 for ε. It will be noticed that, in contrast to Chapter III, only one case is treated in the justification.

V. Co-ascending and co-descending points of the ecliptic

The meaning of the first rule in this Chapter seems to be that half the star's arc of day is subtracted from the right ascension of its *mediatio*, 360° being added if necessary, and the inverse oblique ascension of the result is the ecliptic degree with which the star rises. Let us try to interpret this with the help of Fig. *ii*. $DYXNA$ is the equator; YRM is the ecliptic; S is a star north of the equator and M its *mediatio*, here assumed to be in the first quarter of the ecliptic. The point sought is R of the horizon $XRSB$. The only reasonable arc for determining X is YX. Now $YX = YN - XN$, YN being the right ascension of M, albeit measured from the equinox, and XN is half the excess of the star's arc of day (i.e. over a semicircle, the average arc of day). If, therefore, we assume that the word *faḍl* (excess), or equivalent, slipped from the text in copying, the rule is sound, provided that right ascension may be measured from the equinox — this would perhaps mean that Maslama used another source for Chapter V. This interpretation only works for the other quarters of the

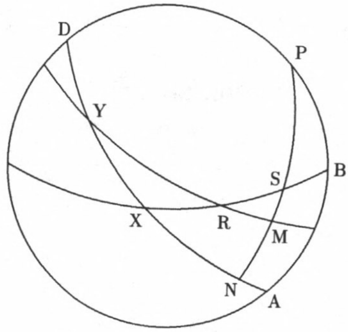

Fig. *ii*

ecliptic if the right and oblique ascensions are measured in the succession of the signs. If they are always measured from the spring equinox, we have perhaps an explanation of *fa-in lam ... dawran* and *si non fuerit unum addes ccclx*[25] as "if it [XN] is not less [than YN], add a rotation" or "if it is not possible, add a rotation", "a rotation" being translated as 360°. For if the equinox Y lies between X and N, YX in the succession of signs $= 360° + YN - XN$. It should be noticed that according to our interpretation a southern star is not treated. In this case the deficiency of the star's day must be subtracted from the ascension of the *mediatio*. Maslama gives no indication of how the arc (or excess) of day is to be found, but Ptolemy gives only this ($BSX/\triangle APN$) and leaves the rest to the reader.

The second rule apparently states that if half the arc of day is added to the right ascension of the *mediatio*, the result is the oblique ascension of the point opposite the point that sets with the star. As Ptolemy says in *Almagest* VIII 5, the

[25]Arabic p. 27,22–28,1; Latin lines 148f.

co-setting point of the equator is the same distance from the co-culminating point as the co-rising point, but on the other side: if Z is the co-setting point (not shown in Fig. *ii*), $ZN = NX$. The second rule, as it stands, is not true: a quadrant would have to be added to the initial sum, i.e. of half the arc of day and the ascension of the *mediatio*. To emend the text to include this would scarcely be appropriate, since it would be easier to subtract a quadrant and so find the ascension of the co-setting point of the ecliptic itself. Two solutions present themselves. One is that the initial sum should be of the ascension of the co-rising degree and the whole of the arc of day. The other, which seems preferable, is that the excess of half the arc of day is taken, as above, instead of half the arc of day itself, and the word *naẓīr* is suppressed or otherwise interpreted.

In the last section, which does not appear in the Latin translation and which may be considered as a supplement to Chapter II, the author explains how to calculate the "declinations" of points on the horizon for every five degrees, the "total declination" being the altitude of the equator, i.e. the complement of the geographical latitude. As with the strictly analogous case of the ecliptic, the rule is

$$\frac{Sin\,5^\circ \cdot Sin\,(\text{alt. equator})}{\frac{1}{2}\text{diam.}} = Sin\,\delta(5^\circ)$$

and so for 10°, etc. Results for the given parameters are given in a table[26], which is mostly accurate, though the minutes are sometimes incorrect. No explanation is offered here of the inaccuracies, which do not appear to have come about by interpolation. The author says that the division of the horizon by declinations is not accurate "as I mentioned to you within the book". This phrase is very similar to that in the Extra-Chapter 24,12–13 (Latin translations, **ABNP** 73–75 and **EF** 55–56)

[26]See Vernet and Catalá, p. 44, and our corrections above on p. 72.

and would be even more similar if *li-mā* in 24,13 were emended to *ka-mā*. Both passages apparently refer to his warning on this count in 22,8 (Latin, **ABNP** 15–16 and **EF** 12); cf. also the end of note 1. The similarity and the colophon in the Arabic (f. 81v) argue for the genuineness of the passage, which might otherwise be doubted because of its non-appearance in Latin.

The last item in the whole Maslama complex in MS Paris, ar. 4821 is a table of twenty-one astrolabe stars registered with longitude and latitude plus *mediatio* and declination for the epoch end of 367 H = 978[27].

[27]See a translation in Vernet and Catalá, p. 45, with annotations by Catalá, pp. 46f., and the edition in Kunitzsch [V], pp. 15ff. ("Type I A").

Correspondence of section numbers and diagrams of the Planisphaerium, *Maslama's notes and the* Propositiones planisperii

<table>
<tr><th colspan="2">section numbers</th><th colspan="2">diagrams</th><th colspan="2">Maslama's notes</th><th rowspan="2">Proposi-tiones[4]</th></tr>
<tr><th>Heiberg</th><th>medieval[1]</th><th>Ar.[2]</th><th>Heiberg p.</th><th>no.</th><th>Heiberg[3]</th></tr>
<tr><td>1</td><td>1</td><td>1</td><td>227</td><td>1</td><td>230: 3</td><td>1 (1)</td></tr>
<tr><td>2</td><td>2</td><td>2</td><td>230</td><td></td><td></td><td>2</td></tr>
<tr><td>3</td><td>3</td><td>3</td><td>232</td><td>2</td><td>232: 14</td><td>3</td></tr>
<tr><td>4</td><td rowspan="4">4</td><td rowspan="4">4</td><td rowspan="4">233</td><td rowspan="4">3</td><td rowspan="4">236: 26</td><td rowspan="4">4</td></tr>
<tr><td>5</td></tr>
<tr><td>6</td></tr>
<tr><td>7</td></tr>
<tr><td>8</td><td>5</td><td>5</td><td>237</td><td>4</td><td>240: 13</td><td rowspan="2">5 & 6</td></tr>
<tr><td>9</td><td>6</td><td></td><td></td><td></td><td></td></tr>
<tr><td>10</td><td>7</td><td>6</td><td>242</td><td></td><td></td><td>7</td></tr>
<tr><td>11</td><td>8</td><td>7</td><td>245</td><td></td><td></td><td>8</td></tr>
<tr><td>12</td><td>9</td><td>8</td><td>246</td><td></td><td></td><td>9</td></tr>
<tr><td>13</td><td>10</td><td></td><td colspan="2">248 (repet.)</td><td></td><td>10</td></tr>
<tr><td>14</td><td>11</td><td>9</td><td>250</td><td>5</td><td>251: 15</td><td>11</td></tr>
<tr><td>15</td><td>12</td><td>10</td><td>251</td><td>6</td><td>252: 12</td><td>12 (6)</td></tr>
<tr><td>16</td><td>13</td><td>11</td><td>253</td><td>7</td><td>254: 4</td><td>13 (7)</td></tr>
<tr><td>17</td><td>14</td><td>12</td><td>254</td><td>8</td><td>255: 18</td><td>14</td></tr>
<tr><td>18</td><td>15</td><td>13</td><td>256</td><td>9, 10</td><td>257: 15</td><td>15</td></tr>
<tr><td>19</td><td rowspan="2">16</td><td rowspan="2">14</td><td rowspan="2">257</td><td rowspan="2">11</td><td rowspan="2">258: 5</td><td rowspan="2">16</td></tr>
<tr><td>20</td></tr>
</table>

[1] These numbers appear in the Latin manuscripts **ED** and some of them also in **KL.** In the Arabic text of Maslama's notes sections 1, 3, 4 and 5 were quoted by numbers and sections 14–19 are identified by their opening and closing phrases. For these sections we have elsewhere used the term "proposition(s)" corresponding to the Arabic *shakl*.

[2] The Arabic version of the *Planisphaerium* in **YT** contains 14 diagrams. In **Y** the places for the diagrams are left blank; in **T** the diagrams are present, but sometimes carelessly drawn. The diagram in § 18 is different in **T** ("Eastern recension") and Hermann ("Western recension") and has different letters.

[3] Indicates the page and line number in Heiberg to which each note is appended in the Latin translation of the *Planisphaerium*.

[4] In brackets are added the numbers of three of Maslama's notes that are reported in the respective *Propositiones*.

Appendix I

Extracts from EF

Here are edited portions or fragments of text added to the *Planisphaerium* in the margins of **EF** (for the small fragments **F** was not used, since it has large illegible sections due to water damage; the three large portions were also edited in Heiberg's *Prolegomena*). It is evident that the translator of **EF** compared Hermann's version with an Arabic manuscript. Most of his additions can be identified in **YT** (for comparison we only cite **Y**, which is available in Anagnostakis' facsimile edition; the text in **T** is virtually identical to **Y**). These translations are partly additions of elements omitted in Hermann's version or more literal renderings of passages given very freely by Hermann or passages of which the Arabic original was different from the "Western" recension used by Hermann, but coincides with the text of **YT**. Similar "corrections" and additions by the translator of **EF** are found in the margins of Maslama's notes nos. 3, 6, 9 and 11 (see the apparatus to the edition, above).

It may be noted that, beyond the marginal notes, there are in **E** numerous additions of single words or short phrases written between the lines, which seem to represent alternative translations of the **EF** translator also. They are not included in the present edition.

E 88v (corresponding to §1, 227:15ff., and partly to **Y** 76v,11–14):

ponemus etiam parallelos sed (?) ut circulos disgressionalem quidem contingentes qui per equalia partitur (?) equatorem, partitus (?) tamen (?) inequaliter

E 88v (not in **Y** 77r,1, nor in Hermann §1, 227:26):

spera quippe superficiata, et hic apparet ... (last word illegible in the binding)

E 89v (different from **Y** 78v,9ff. and Hermann §2, 230:19ff.):

Dico quod duo puncta Z et H similia sunt in potencia duobus punctis diametralibus in spera id est paralleli altrinsecus ab equatore super hec in spera facti sunt equales

E 90r = **Y** 80r,10–11 (§4, 233:2–4):

ut sciamus quod elevationes invenientur numero convenienti[1] ei quod apparet in spera recta et declivi

E 90r = **Y** 81v,5–6 (§4, 235:1; not in Hermann):

de cxx[2] partibus diametri

E 90r = **Y** 81v,11–13 (§5, 235:6–8):

vel que est longitudo parallelorum[3] qui secant ex orbe signorum ab utraque parte notarum tropicorum xxx gradus

E 91r = **Y** 84r,12–13 (§8, 238:12–14):

vel quorum cuiuslibet si reperiamus quantitatem erit ut quantitas arcus BM. Ex hoc etiam colligemus quod intendimus de elevationibus

E 91r (added to the text in Heiberg §8, 239:4, not identified in **Y**; cf. Heiberg, p. CLXXXVIII):

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[1] convenientes **Ar.**

[2] 60 **Ar.**

[3] longitudo parallelorum] longitudo inter equatorem et circulos equidistantes equatori **Ar.**

Aliter: et hoc quia, cum ducetur EK in id quod relinquitur de duplo EF cum quadrato ET, proveniet quadratum EK, quia etiam TK equatur BT, que subtenditur angulo recto, cuius alterum latus ET et reliqua circuli minoris semidiameter est.

E 91r (representing part of Y 85r,ult.–85v,3; not in Hermann, §8, 239:26ff.):

et de totidem duorum rectorum partibus erit 55 partes et 40 minuta

E 91v = Y 86r,5–7 (§9, 240:27; not in Hermann):

Ex premissis ergo, cum explanabitur nobis quantitas arcus BM equatoris, sciemus tempora cum quibus elevatur unumquodque signorum predictorum in spera recta

E 91v = Y 86v,9–11 (§9, 241:16; not in Hermann):

habebit etiam 115 gradus et 28 minuta de 360 partibus duorum rectorum

E 91v = Y 86v,15–16 (§9, 241:21; not in Hermann):

quam ostendimus esse 27 gradus et 50 minuta

E 91v = Y 87r,4–5 (§9, 241:27; not in Hermann):

videlicet 32 gradus et 16 minuta et hoc consonat illi quod diximus in spera

E 91v = Y 87r,10–11 (§10, 242:4; not in Hermann):

super orizontem qui describitur cum circulis quorum longitudines ab equatore sunt que dicte sunt

E 92r = Y 89r,10–11 (§11, 245:12; not in Hermann):

et erit 37 gradus et 30 minuta de 360 partibus duorum rectorum

E 92v (representing part of Y 90r,15–17; not in Hermann, §12, 247:10):

Sed de totidem duorum rectorum partibus triangulus *ECY* 17 gradus et 16 minuta

E 93r = Y 91v,10–11 (§13, 249:8; not in Hermann):

Quoniam fuit elevatio virginis in 36 temporibus et 27 minutis

E 93r = Y 93r,4–5 (§15, 251:27):

nullo (?) habito respectu ad equatorem

E 93r and F 218r = Y 93r,11–16 (§15, 252:4–9; cf. Heiberg, p. CLXXXVIII):

Alia translatio: erit nota *K* nadair polo circuli signorum in potentia. Et manifestum est quod hoc erit secundum quod aperiuntur, quoniam circulus transiens per hanc notam et per duas oppositas notas secundum diametrum in circulo signorum secat etiam equatorem in duo media, et erunt isti circuli positi loco circulorum maiorum stantium super orbem signorum orthogonaliter.

E 93v = Y 93v,12–13 (§16, 252:25–26):

et propter hoc transeunt isti circuli per longitudinem *EM* et *EN*[4]

E 93v (corresponding to §16, 253:6–8; not in Y — the entire passage 253:4–8 is absent in Y):

secundum quod videtur in spera solida

E 93v = §16, 253:15–17 (not in Y 94r,1):

et adiungam *B* cum *R*

E 94r (different from both Y 95r,1–3, and Hermann §17, 255:11–13):

Vel sic: et ponam *LN* in *MC* communem. Erit ergo linea *NM* in *MC* cum *LN* in *MC* maior linea *CL* in *NL* cum *LN* in *MC* et linea *NM* in *MC* cum *LN* in *MC*; equatur *LM* in

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[4]Translating the same Arabic text as Hermann, but reading *tamurru* instead of **YT**'s and Hermann's *tursamu*.

MC et similiter CL in NL cum CM in NL; equatur LM in NL. Fiet ergo LM in MC maior ML in LN; ergo MC maior est NL.

E 94r = **Y** 95r,9 (§18, 255:20):

quia cadit pars eius in sectionem que non apparet[5]

E 94r (a rendering by the **EF** translator of Hermann's §18, 256:6–7, which is different from the wording in **Y** 95r,ult.–95v,1):

id est: erit circulus NYQ circulus occultus in perpetuum in lamina

E 94r (a rendering by the **EF** translator of Hermann's §18, 257:5ff., both being completely different from the text in **Y** 95v,1ff.):

ergo circulus qui lineatur loco circuli qui est super lineam TL transiens per notam Y transibit per punctum C cuius longitudo ab equatore est arcus GL et iam transivit per punctum Y et dividit circulum NYQ in duos arcus similes illis

E 94v = **Y** 96r,7–8 (§20, 258:8–9):

id est quorum quidam sunt equidistantes equatori

E 95r = **Y** 96r,10–11 (not in Hermann):

id est: erunt omnes circuli meridiei linee recte

E 95r = **Y** 96r,ult.–96v,1 (§20, 258:23):

et erunt reliqui circuli inequales

E 95v and **F** 219v = **Y** 96v,1–97r,4 (§20, 258:23–259:16; at the end this version adds its own explicit; cf. Heiberg, pp. CLXXXVIII and CLXXX, note 1):

Alia translatio: et oportet ex hoc ut sit possibile in positionibus que reperiuntur cum comparatione ad equatorem signari stellas quamvis non lineentur omnes circuli descripti. Et secabimus secundum proportiones circulorum equidistantium

[5]The same text as in Hermann, but here completely rendered, whereas Hermann has it extremely abbreviated.

equatori et cum divisione equatoris solius. Et in positionibus que reperiuntur cum comparatione ad orbem signorum non est possibile hoc, sed oportet ut lineentur omnes circuli aut plures eorum ad demonstrandum locum ubi oporteat poni stellas. Et de iustioribus rebus est ut compleamus in utraque harum duarum notarum quod fecimus in spera ut ponamus circulos qui reperiuntur causa equatoris, illos qui sunt meridiei et illos qui sunt equidistantes equatori et circulorum causa circuli signorum repertorum, illos qui sunt propinquiores[6]. Et si non possint hec omnia lineari in lamina, reliquum est ut lineentur circuli transeuntes per duos gradus aut tres aut sex, cum sit hec descriptio media, quia isti numeri sunt communicantes cum xxx, numero graduum signorum, et cum xxiiii, numero longitudinis equatoris ab utroque tropicorum fere, donec lineentur circuli tropicorum et circuli meridiei qui transeunt per signa. Et non erit in longitudinibus secundum aliud exemplum repertis diversitas, si deus voluerit.

Explicit liber superficiate spere ptholomei correctus a meslem filio dantis gratias quod est admeti.

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[6] *illos qui sunt propinquiores*] Hermann (259:8–9) has *...ad exemplum fiant, quantum fieri potest, propinquum Egipto*; cf. also Drecker, p. 278, Sinisgalli and Vastola, p. 255. However the Arabic text (**Y** 96v,12 and identically **T**) simply says: *ʿalā mithāl mā ʿalayhi al-ukar al-maḍrūba*, "as [the matter is with] the hammered globes". Both Latin translators seem to have followed corrupt Arabic copies. There is no reference in the text to Egypt (*Miṣr*), as wrongly maintained — following Hermann — by Neugebauer [II], p. 871 with note 7. Heiberg reports in the apparatus of his edition (p. 259, *ad* l. 9) that **A** here has the marginal note *in alio: illi loco*, and **B** adds (to *egipto*, in the text) also the words *illi loco*; this seems to reflect another Arabic variant, which, however, is again different from the reading in **YT**.

Appendix II

Propositiones planisperii

The *Propositiones* comprise a summary of the sixteen propositions of the *Planisphaerium* (in the medieval numbering) including three quotations from Maslama's notes, the whole apparently written in Latin. This text is always accompanied by two short texts, *Si a termino* ... and *Radicem* .... Together they normally appear after the *Planisphaerium* — in the known *Planisphaerium* manuscripts they are missing only in **ABNP** and **MSX** — and are found, separately, in MS Oxford, Corpus Christi College 224, ff 139r–141r (here **O**). In **DEOF** the two short pieces come after the *Propositiones* and in **KCWL** before it. Each of these two groups has other common characteristics. The numbering of the propositions occurs only in **DE**; it has been retained in this edition. **L** has a different numbering. It is not known how many authors are involved in the writing of the three texts.

When a *propositio* reports what the aim of the relevant section of the *Planisphaerium* is (e.g. to find an ascension or to place a circle), this is expressed, without giving the procedure, by an accusative and infinitive. Sometimes further results or procedures are given after the phrase *unde manifestum* [*est*]. When Ptolemy presents a calculation, he does so by means of an example, but the *Propositiones* gives instead a verbal description of the procedure. With the exception of the rule given in P.4 such a passage is always introduced by the word *Regula*. There are no numbers in the *Propositiones*. Three Maslama notes are reproduced in substance: 1 (P.1), 6 (P.12) and 7 (P.13). In each case the content is not of mathematical interest, but emphasizes the application of the construction described. It is curious that both Maslama's notes 6 and 7 are quoted, for their substance is similar.

K pp. 185–189, C 217rb–217vb, D 10v–12v, E 95r–96r, W 10v–11v, L 57r–58r, O 139r–140v (F 219v–221v, not reported here)

Incipit prima propositio planisperii

Quoslibet duos circulos equidistantes recto in spera corporea circulumque declivem eos e regione contingentem in plano collocare. Unde manifestum est circulum rectum a circulo declivi in plano quoque per equalia secari atque secare illum potentialiter per equalia.

Maslem: si ad quantitatem declinationis quorumlibet oppositorum graduum zodiaci arcus abscidantur a circulo recto ex utraque parte linee meridiane et per puncta sectionum due linee ducantur a puncto occidentali donec secent lineam meridianam, illa duo puncta sectionum meridiane linee metientur quantitatem circulorum potentialiter equidistantium recto qui scilicet transeunt per illos gradus oppositos.

Secunda

Omnis linea recta ducta per centrum circuli recti ferit puncta circuli declivis sese potentialiter respicientia.

Tercia

Si linea ducta a puncto unius sectionum orizontis et zodiaci protrahatur per centrum circuli recti, necesse est lineam illam

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1 Incipit ... planisperii] *om.* KCWL, i E, Capitula planispherii sive theoreumata L (*et id. in marg.*), Propositiones Planisphaerii Ptolemei cum quibusdam ... O (*rec. manu*) *2* duos] cap *add. et del.* L *2* recto] rectos C *3* eos e regione] eosque repone O *4* Unde manifestum] patet L *4* circulum] circulumque DEO *7* Maslem] Maslen DEO *8* si ad ... arcus] *om.* C *9* et] *om.* KCWL *9* sectionum] scicionum O *10* due linee] linee due KWL, linee C *11* sectionum] siccionum O *11* ducantur a ... linee] *om.* C *13* scilicet] *om.* O *13* transeunt] transit DEO *14* Secunda] *om.* KCWO, ii E *15* linea recta] recta linea O *15* recta ducta] *corr. ex* ducta recta D *15* ferit] facit W *17* Tercia] *om.* KCWO, iii E *18* linea] recta *add.* C *19* per] *om.* C *19* est] per *add.* C transire per punctum sectionis relique. Unde manifestum est quemlibet circulum, qui secat per medium circulum rectum, zodiacum etiam per equalia potentialiter secare et convertitur.

Quarta

Inter semidiametra quorumlibet circulorum potentialiter equidistantium recto semidiametrum circuli recti sub continua proportionalitate collocatam esse necesse est. Eritque illarum semidiametrorum proportio maioris ad minorem tanquam proportio corde arcus qui constat ex quadrante circuli recti et arcu declinationis illorum circulorum potentialiter equidistantium recto ad cordam arcus residui de semicirculo circuli recti. Unde manifestum est quod si diametros circuli recti fuerit nota fueritque nota declinatio circulorum potentialiter equidistantium recto, illorum quoque semidiametri erunt note. Que pariter accepte complent diametrum circuli declivis eos e regione contingentis.

Quinta cum sexta

Quarumlibet iiii^{or} portionum zodiaci equalium, quarum due contingunt unum punctum equalitatis, relique due reliquum, cum equis arcubus circuli recti elevantur in plano secundum situm recti orizontis, qui est in potentia linee recte in plano. Unde manifestum ad scientiam elevationum in plano unius earum scientiam sufficere indagare.

Regula. Si <a semi>diametro circuli equidistantis exterioris, cuius declinatio est tanquam declinatio portionis meridiane

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21 quemlibet] quamlibet L *21* circulum rectum] rectum circulum L *22* potentialiter secare] secare potentialiter DEO *23* Quarta] *om.* KCWO, iiii E *24* Inter] Item L *24* semidiametra] semidiametros C, semidiametrum O *24* quorumlibet] quodlibet O *25* semidiametrum] semidiametro O; a *add. et del.* L *26* illarum] illorum DEOL *27* minorem] minus L *27* tanquam] sicut L *28* qui] *om.* C *29* illorum] istorum O *29* potentialiter] *om.* L *31* Unde manifestum] patet L *31* diametros] diametrum L *31* nota] notum L *32* fueritque] fuitque O *33* ad cordam ... recto] *om.* KCW *33* note] nota L *33* Que] qua L *34* eos] ea L *36* Quinta cum sexta] *om.* KCWO *39* secundum] *om.* KW *43* Regula] *om.* O *43* Si] a semicirculo vel L *43* diametro] diametri DEO *43* circuli] recti *add.* D *43* exterioris] thoris O *44* cuius] eius O desinentis in punctum equalitatis cuius queritur elevatio in plano, subtrahatur semidiametros interioris equidistantis cuius eadem est declinatio residuique dimidium ducatur in 120 productumque dividatur per distantiam centri circuli recti a centro zodiaci, exibit corda dupli arcus quesite elevationis in plano. Vel sic. Si quadratum semidiametri circuli recti dividatur per semidiametrum exterioris equidistantis, cuius declinatio est a circulo recto tanquam declinatio portionis inchoate a puncto equinoctialis cuius queritur elevatio in plano secundum situm spere recte, et quod exierit detrahatur a semidiametro divisore, remanebit linea cuius medietatis proportio est ad distantiam centrorum circuli recti et circuli declivis tanquam proportio corde dupli arcus quesite elevationis ad diametrum circuli recti.

Septima

In omni declinatione quantum elevatio portionis que est circa punctum vernale decrescit ab elevatione eiusdem in spera recta, tantumdem crescit elevatio portionis opposite. Unde manifestum est differentiam elevationum unius quadrantis, qui est inter punctum vernale et alterutrum tropicorum, in spera recta et orizonte declivi esse tanquam dimidiam differentiam diei equinoctialis et diei minimi in eadem declinatione.

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45 desinentis] desinente MSS *45* elevatio] declinatio KW, elevatio vel declinatio L *46* queritur elevatio ... cuius] *om.* C *48* circuli recti] *corr. ex* recti circuli W *50* sic] *om.* KCW *51* equidistantis] equidem (?) O *52* tanquam] sicut L *53* equinoctialis] equalitatis KWL *54* a] etiam W *55* remanebit] removebit K *55* medietatis] medietas W *57* tanquam] sicut L *59* Septima] *om.* KCWO, vii E *60* declinatione] vel elevatione L; differentiam *add.* O; qua habundat dies maximus *add. et del.* O *61* vernale] universale C *61* decrescit] descendit C; et *add.* C *61* eiusdem] equidem O *62* tantumdem] tantum CL *62* opposite] vel opposita *add.* L *63* manifestum est] patet C *63* differentiam] distantiam DEO *63* elevationum] elevationis C *65* dimidiam] disiunctam O *65* differentiam] distantiam DEO *66* diei] die L

Octava

In omni declinatione differentiam qua habundat dies maximus a die equinoctiali investigare. Unde manifestum est quaslibet differentias arcuum dierum in plano in spera corporea constare ex partibus numero equalibus; cuiuslibet quoque portionis elevationes equales esse in plano et in spera corporea.

Regula. Si distantia centrorum circuli recti et zodiaci ducatur in 120 partes et productum dividatur per distantiam centrorum orizontis et circuli recti, exibit corda arcus qui est tanquam differentia qua habundat arcus diei equinoctialis ab arcu diei minimi.

ixª

In omni declinatione in qua altitudo poli fuerit nota, elevationem cuiuslibet portionis inchoate vel terminate in puncto equalitatis investigare.

Regula. Si a quadrato semidiametri orizontis subtrahatur quadratum distantie centrorum circuli recti et orizontis residumque dividatur per semidiametrum exterioris equidistantis, cuius declinatio a circulo recto est tanquam declinatio portionis cuius queritur elevatio, et quod exierit subtrahatur ab eadem semidiametro divisore, remanebit linea, medietatis cuius proportio est ad distantiam centrorum orizontis et circuli recti tanquam proportio corde dupli arcus qui est differentia elevationum eiusdem portionis in circulo recto et orizonte declivi ad diametrum circuli recti. Ducatur ergo primum in quartum etc. Vel sic: si a <semi>diametro equidistantis exterioris, cuius eadem est declinatio que et portionis cuius queritur elevatio,

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67 Octava] *om.* **KCW**, viii **E** *68* qua] quam **W** *68* differentiam qua habundat] qua habundat differentiam **C** *69* est] *om.* **C** *69* quaslibet] qualibet **C** *70* arcuum] ancuum (?) **C** *71* quoque] etiam **L**, quousque **O** *71* portionis] elevationis *add.* **DEO** (*corr. ex* elavationis **DE**) *73* Regula] *om.* **O** *74* partes] *om.* **KCWL** *75* corda] cordas **C** *76* tanquam] sicut **L** *76* qua] que **L** *76* equinoctialis] *om.* (*loc. vac.*) **O** *78* ixª] *om.* **KCWO**, ix **E** *79* poli] *om.* **KCW** *79* fuerit] fuit **O** *80* elevationem] elevatione **DEO** *82* Regula] *om.* **O** *83* circuli] *om.* **C** *84* equidistantis] equidem **O** *87* eadem] eodem **DEO** *87* semidiametro] semidiametri **L** *87* cuius] eius **O** *88* distantiam] differentiam **L** *89* tanquam] sicut **L** *89* proportio] dupli *add.* **DEOKW** *89* qui] que **L** *89* differentia] *om.* **KCW** *92* equidistantis] equidem **O** subtrahatur semidiametros interioris equidistantis cuius eadem est declinatio, remanebit linea, medietatis cuius etc. ut prius. Vel sic: si quadratum semidiametri circuli recti dividatur per semidiametrum equidistantis exterioris et quod exierit subtrahatur ab eadem semidiametro divisore, remanebit linea etc. ut prius.

x^{a}

In omni declinatione in qua altitudo poli fuerit nota, elevationem cuiuslibet portionis zodiaci reperire.

xi^{a}

Proposito quolibet circulo australi cuius nota sit declinatio a circulo recto, intra eum circulum rectum in plano collocare.

xii^{a}

Punctum potentia respicientem polum zodiaci reperire in plano.

Maslem. Omnis circulus transiens per punctum potentia respicientem polum zodiaci et per gradus eius oppositos si per corpus stelle transeat, per gradum stelle transibit et econtrario; secabit etiam circulum rectum per equalia. Item omnis linea recta ducta per centrum circuli recti si per corpus stelle transeat, transibit etiam per gradum zodiaci cum quo stella transit lineam meridianam, et hoc quoque convertitur.

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94 interioris] exterioris **K** *95* que et ... declinatio] *om.* **O** *97* equidistantis] equidem **O**; *seq. ras.* **O** *98* eadem] eodem **DEO** *99* linea] *om.* **KCW** *99* Vel sic] si a diametro ... prius:: *om.* **L**; suo collocare *add. et del.* **C** *100* x^{a}] *om.* **KCWO**, x **E** *101* poli] *om.* **KC** *101* nota] elie *add. et del.* **W** *102* elevationem] elevationes **L** *103* xi^{a}] *om.* **KCWO**, prima secundi **L** *104* Proposito] Propōtio **K**, Proportio **W** *104* nota] nata **C** *104* declinatio] de circulo **C** *105* eum] cum **C**; intra eum *repet.* **O** *106* xii^{a}] *om.* **KCWO**, xii **E**, 2^{a} **L** *109* Maslem] M̄ **KW**, Maslen **DEO**, M^{u} **C**, M **L** *109* transiens] *om.* **K** *109* potentia] ponat **C** *111* per gradum stelle transibit] transibit per gradum stelle **C** *112* equalia] equa **KCWL**, *seq. ras.* **K**, ib (?) *add. et del.* **L** *112* Item] Itemque **KCW** *113* recta] *om.* **KCWL** *114* gradum] gradus **K** *115* transit] transeat **KCW** *115* et hoc] etiam **C** xiii^{a}

Quoslibet circulos equidistantes zodiaco in plano collocare. Unde colligitur simili ratione circulos equidistantes orizonti in plano posse designari.

Maslem. Si circulus equidistans zodiaco, cuius distantia metitur latitudinem, collocetur in plano, deinde per punctum potentia respicientem polum zodiaci et per gradum stelle ducatur arcus circuli potentia zodiacum actu circulum rectum secantis per medium, punctus sectionis ubi arcus iste secat illum equidistantem zodiaco locum stelle in planisperio denotabit.

xiiii^{a}

Quorumlibet circulorum equidistantium zodiaco designatorum in plano centra diversa esse necesse est.

xv^{a}

Circuli equidistantis zodiaco et circuli semper latentis equidistantis circulo recto se secantium punctum sectionis reperire in plano. Unde manifestum quoslibet circulos equidistantes zodiaco a circulis equidistantibus circulo recto quos secant, sive sint latentes sive non, similes arcus in plano et in spera corporea resecare.

xvi^{a}

Circulus equidistans zodiaco per polum australem transiens, habet designari in plano per lineam rectam perpendicularem

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116 xiii^{a}] *om.* KCWO, xiii E, 3^{a} L *118* circulos equidistantes] equidistantis L *118* orizonti] orizontes C *119* posse designari] et collocare vel designari posse L *120* Maslem] M̄ K, Maslen DEO, M^{u} C, M WL *120* distantia] distantie L *121* latitudinem] longitudinem O *121* collocetur] locetur L *122* potentia] ponat C *122* gradum] gradus KW *123* circuli] *om.* O *123* potentia] ponat (?) C *123* actu] arcu DO; actum KCW; vel acti L *124* secantis] secans C *124* punctus] punctum C *124* iste] hic L *125* denotabit] pernotabit KCW *126* xiiii^{a}] *om.* KCWO, xiii^{a} D, xiiii E, 4^{a} L *127* circulorum] *om.* KCW *127* designatorum] designatur KW *128* centra diversa] diversa centra KCW *129* xv^{a}] *om.* KCWO, xv E, 5^{a} L *131* recto] *om.* O *132* manifestum] patet L; est *add.* E *132* quoslibet circulos] circulos quoslibet L *133* circulo] *om.* L *133* circulo recto] *corr. ex* circulis rectis O *136* xvi^{a}] *om.* KCWO, xvi E, 6^{a} L *137* equidistans] equidistantis L diametro equinoctialis circuli eductam a puncto incisionis eius in quod diameter eius equidistantis zodiaco productus incidit, eductam inquam in circumferentiam circuli representantis equidistantem recto. Unde manifestum quod medius omnium eorum, quos secat hic equidistans zodiaco transiens per polum equinoctialis, invicem quoque hic equidistans recto per polum zodiaci transire necesse est vel habet.

Expliciunt propositiones.

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K p. 184, C 217ra–b, D 12r–v, E 96r–v, W10r–v, L 56v–57r, O 140v–141r (F 221v–222r, not reported here)

Si a termino unius diametri circuli recti ducatur linea per centrum circuli declivis usque ad circumferentiam, necesse est eam includere cum eadem diametro arcum duplum arcui declinationis eiusdem circuli declivis. Proportio quoque distantie centri circuli declivis a polo eiusdem ad distantiam eiusdem poli a centro circuli recti est tanquam proportio semidiametri circuli declivis ad semidiametrum circuli recti.

Ratio. Sit enim *ABGD* circulus rectus descriptus circa centrum *E*; linea vero *DFC* metiatur quantitatem circuli equidistantis exterioris; linea vero *DH* metiatur quantitatem equidistantis interioris; linea *DKL* transeat per centrum circuli

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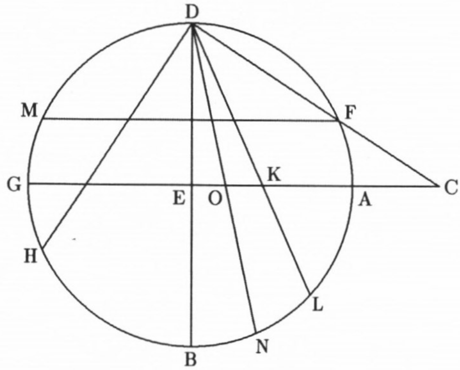
139 circuli] *om.* **L** *139* incisionis] visionis **KCW** *140* quod] quo **C**, quem **W** *140* diameter] diametrum **L** *140* eius] ipsius **KWL** *141* eductam] eductum **DE**, rductum̄ **C** *141* circumferentiam] periferiam **L** *141* representantis] *corr. ex* representeis **K**, representatur **C**, representantem (*corr. in* (?)) **O** *142* manifestum] patet **L**; est *add.* **C** *142* medius] melius **DEO** *143* transiens] pertransiens **C** *144* equinoctialis] equinoctialem **DO**, *corr. ex* equinoctialem **E** *144* equidistans] distans **O** *145* est vel] *om.* **DEO**, necesse est vel *om.* **C** *145* est vel habet] habet vel est **L** *146* Expliciunt propositiones] *om.* **KCWL** *1* ducatur] *corr. ex* (?) **K** *1* linea] recta *add.* **O** *2* circuli] *om.* **L** *2* circumferentiam] periferiam **L** *3* cum] in **L** *4* declivis] *repet. et del.* **K** *4* quoque] differentie *add. et del.* **K** *5* circuli] *om.* **L** *5* distantiam] differentiam **L** *8* Ratio] *om. (loc. vac.)* **O** *8* *ABGD*] *ABDG* **C** *8* circa] *corr. ex* supra **E** *9* *E*] C **C** *9* vero] *om.* **C** *9* *DFC*] *DFT* **DEO** *11* exterioris ... equidistantis] *om.* **C** *11* linea vero ... interioris] *om.* **DO**, *marg.* **E** (dist *add. et del. post* metiatur); interioris:: *om. (loc. vac.)* **W** *11* circuli] *om.* **L** declivis in puncto K. Dico quod arcus LB duplus est arcui FA. Protrahatur enim FM equidistans diametro AG. Quoniam ergo KDC angulus equalis est angulo KCD, qui equalis est angulo MFD, qui equalis est angulo BDH, erit arcus LF equalis arcui BH. Set arcus FD est equalis arcui DM: ergo arcus LB est equalis arcui MH, qui est duplus arcui FA.



[Fig. 16]

Secunda pars sic probatur. Dividatur arcus LB in duos arcus equales, scilicet LN et NB; ducaturque linea DON, quam necesse est transire per polum circuli declivis in puncto O, eo quod perfectio declinationis cuiusque circuli declivis est a circulo recto tanquam declinatio poli circuli declivis a circulo recto. Quoniam ergo angulus KDE divisus est per duo equalia, erit proportio KO ad OE tanquam proportio KD semidiametri circuli declivis ad ED semidiametrum circuli recti.

13 protrahatur] protrahitur **C** *14* ergo] *om.* **KCW** *14* *KDC*] *KDT* **DEO**, *KLD* **KW** *14* equalis est[1]] est equalis **KCW** *15* equalis est[2]] est equalis **KW** *15* equalis est] est equalis **KEW**, est equus **L** *15* *MFD* qui angulo] *marg.* **E**, *om.* **O** *16* equalis] a linee *HD* alie enim equatur (?) *marg.* **C** *16* est] *om.* **C** *17* arcui] *om.* **K** *17* *FA*] *FH corr. in FA* **E**, *FH* **LO** *19* arcus equales] equos **L** *19* quam] quod **DEO** *20* circuli declivis] declivis circuli **KW**, declivis circulis **C** *20* *O*] *om.* **C** *22* tanquam] sicut **L** *23* tanquam declinatio ... recto] *om.* **C** *23* *KDE*] *corr. ex KED* **K**, *KDC* **C** *23* est] *supra* **O** *24* tanquam] sicut **L** *25* *ED*] *EO* **O** *26* recti] constat planisper *add.* **O**

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Radicem planisperii sic colligere possumus.

Constat planisperium nichil aliud esse quam planitiem equinoctialis circuli quantumlibet extensam. In hac autem planitie omne punctum spere corporee preter alterutrum polorum representari est possibile. Modus vero representandi is erit: a polo quem in plana spera latere malumus, id est australi, ad quodvis punctum in spera corporea datum tendatur linea et donec cum planitie equinoctialis concurrat protendatur, punctumque in quo hec incidit in plano datum punctum in spera potentialiter ostendit.

Comments

In the following, first a few minor deviations of the *Propositiones* from the contents of the *Planisphaerium* are noted; then the verbal formulae are given in modern form; then a few remarks are made about the Latin style; finally the two smaller texts are considered.

In P.1 the condition that the equator-parallels be equidistant from the equator is omitted from the opening sentence. Much *Planisphaerium* material is here ascribed to Maslama (note 1).

In P.13, on placing the zodiac-parallels, it is added that the same procedure may be applied to the parallels to the horizon. Probably the author took this addition from the last sentence of Maslama's note 7, which he cites without this passage.

P.15 is on the intersection of the representations of a zodiac-parallel and an always hidden equator-parallel. The Arabic

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27 sic] ex his **KCWL** *28* Constat planisperium] tum (?) **O** *29* extensam] extensa **C** *30* omne] omnem **C** *31* erit] est **L** *35* punctumque] punctus **C** *35* hec] hoc **O** *35* spera] corporea *add.* **L** text of the *Planisphaerium* considers a single equator-parallel, no doubt the greatest always-invisible circle, but Hermann's Latin sometimes has circles (plural) instead. The *Propositiones* rightly says that the theorem applies to all equator-parallels, *sive sint latentes sive non*. This appears to be the one original remark in the work.

P.16. Evidently, *eductam inquam in circumferentiam circuli representantis equidistantem recto* refers to the equator-parallel that appears in the diagram and argument of the *Planisphaerium*, the equator-parallel taken by Maslama to be the [greatest] always-invisible circle. The last sentence is apparently taken from the end of Maslama's comment on the passage (note 11 — see above).

The formulae in the *Propositiones* determine the radii of the equator-parallels of given declination and the right and oblique ascensions. In these matters Ptolemy considers opposite points of the zodiac together. Let us here represent the semidiameters of the exterior (southern) and the interior (northern) equator-parallels of equal declination by r_1 and r_2 respectively, the semidiameter and diameter of the equator by r and d, the distance between the centres of the equator and the zodiac by Z, the semidiameters of the zodiac and horizon by r_z and r_h, the distance between the centres of equator and horizon by H, and the chord function by Cd, where $Cd\ x = 2r \sin \frac{1}{2}x$. The formula $r_1 \cdot r_2 = r^2$ proved in §4 (P.4) is used constantly. In P.5-6 two formulae are given to find the right ascension, α:

(i) $\frac{1}{2}(r_2 - r_1) \times \frac{120}{Z} = Cd\ 2\alpha$,

(ii) $r_2 - \frac{r_2^2}{r} = y$ such that $\frac{1}{2}y : Z = Cd\ 2\alpha : d$.

In §8 Ptolemy proves the equivalent of the second rule and then gives an "easier" way, which consists of a shorter demonstration of the same rule — in the first method he had unnecessarily introduced $r_z^2 - Z^2$ into the argument (cf. $r_h^2 - H^2$ below).

The difference of the equinoctial and shortest day is given in P.8 by

$$\frac{Z\times 120}{H} = Cd \text{ (eq. day - min. day)}$$

as in the *Planisphaerium*.

In P.9 three rules are given for finding the ascensional difference, Δ, i.e the difference between the right ascension and the ascension in a given horizon:

(i) $r_2 - \frac{r_h^2 - H^2}{r_2} = y$ such that $\frac{1}{2}y : H = Cd\,2\Delta : d$

(ii) $r_2 - r_1 = y$ etc.

(iii) $r_2 - \frac{r^2}{r_2} = y$ etc.

In the *Planisphaerium* the $r_h^2 - H^2$ in (i) is immediately resolved into 3600 (i.e. r^2), so that (iii) is given there.

In the main the syntax and style of the *Propositiones* is typically Latin, though there are a few unusual constructions such as *linea, cuius medietatis* ... (twice in P.9). *Circulus rectus*, *punctum equalitatis*, *portio*, *tanquam* are regularly used for "equator circle", "equinox", "arc", "equal to". *Horizon rectus* and *horizon declivis* are used in P.5-6 and P.7, respectively, for "*sphaera recta*" and "*sphaera obliqua*". Finally, we note the odd term *e regione contingens* (P.1, 4) used of tangent circles.

Of the two small texts, *Radicem planisperii* ... is a general statement of the principles of stereographic projection. In contrast, *Si a termino* ... states and proves a particular result: that (in Fig. 16) if BL is twice the obliquity of the ecliptic, then K is the projection of the pole of the zodiac. No direct source for this is known; suffice it to say here that the theorem was also known in the Arab world, for the equivalent result (without the proportion) for the horizon is stated in al-Sijzī's treatise on all forms of the astrolabe (MS Istanbul, Ahmet III 3342, f. 134v).

The proof may be summarized:

$$\angle KDT = \angle KTD \text{ [I 5]} = \angle MFD \text{ [I 29]} = \angle BDH \text{ [III 27]}.$$

Thus arcs LF and BH are equal [III 26]. Therefore, arcs FD and DM also being equal, arc BL is equal to arc MG, i.e. to double arc FA. The proportion follows immediately by VI 3. The diagram appears in only two manuscripts, **C** and **L**. In **C** it is essentially as reproduced here, but in **L** it appears reflected left-to-right.

Bibliography

Anagnostakis, C., *The Arabic Version of Ptolemy's Planisphaerium*, Ph.D. thesis, Yale University 1984 (published in book form by University Microfilms International, Ann Arbor, Michigan, in 1986).

Blachère, R., *Livre des catégories des nations*, Paris 1935.

Burnett, C. S. F., 'Arabic into Latin in Twelfth Century Spain: the Works of Hermann of Carinthia', *Mittellateinisches Jahrbuch* 13 (1978), 100-134.

Drecker, J., 'Das Planisphaerium des Claudius Ptolemaeus', *Isis* 9 (1927), 255-278.

DSB = *Dictionary of Scientific Biography*, ed. C. C. Gillispie, I-XVIII, New York 1970-1990.

Heiberg, J. L. (ed.), Ptolemaeus, *Opera*; *Planisphaerium*: vol. II, Leipzig 1907, pp. 227-259; cf. also the Prolegomena, pp. XIIf.; CLXXX-CLXXXIX.

Ibn al-Nadīm, *Kitâb al-Fihrist*, ed. G. Flügel, I-II, Leipzig 1871-1872.

Jordanus, see Thomson.

Al-Khwārizmī, *Tables*, see Suter and Neugebauer [II].

Kunitzsch [I] = Kunitzsch, P., 'Letters in Geometrical Diagrams, Greek-Arabic-Latin', *Zeitschrift für Geschichte der Arabisch-Islamischen Wissenschaften* 7 (1991/92), 1-20.

Kunitzsch [II] = idem, 'The Role of Al-Andalus in the Transmission of Ptolemy's *Planisphaerium* and *Almagest*' (to appear in the *Proceedings* of the Fifth International Symposium on the History of Arabic Science, Granada, 30 March - 4 April 1992).

Kunitzsch [III] = idem, 'The second Arabic manuscript of Ptolemy's *Planisphaerium*' (to appear in *Zeitschrift für Geschichte der Arabisch-Islamischen Wissenschaften*).

Kunitzsch [IV] = idem, *Glossar der arabischen Fachausdrücke in der mittelalterlichen europäischen Astrolabliteratur*, Göttingen 1983 (Nachrichten der Akademie der Wissenschaften in Göttingen, Phil.-hist. Kl., 1982, 11).

Kunitzsch [V] = idem, *Typen von Sternverzeichnissen in astronomischen Handschriften des zehnten bis vierzehnten Jahrhunderts*, Wiesbaden 1966.

Lemay, R., 'De la scholastique à l'histoire par le truchement de la philologie: itinéraire d'un médiéviste entre Europe et l'Islam', in *La diffusione delle scienze islamiche nel medio evo europeo*, Convegno internazionale, Roma, 2-4 ottobre 1984, Rome 1987, pp. 399-535.

Lorch, R., 'Ptolemy and Maslama and the Transformation of Circles into Circles in Stereographic Projection' (forthcoming).

Margoliouth, G., *Catalogue of the Hebrew and Samaritan Manuscripts in the British Museum*, 4 vols., London 1899-1935.

Nallino, C. A. (ed.), *Al-Battānī sive Albatenii opus astronomicum*, I-III, Milan 1899-1907.

Neubauer, A., *Catalogue of the Hebrew Manuscripts in the Bodleian Library*, Oxford 1886.

Neugebauer [I] = Neugebauer, O., *The Astronomical Tables of al-Khwārizmī*, Copenhagen 1962.

Neugebauer [II] = idem, *History of Ancient Mathematical Astronomy*, I-III, Berlin etc., 1975.

Ptolemy, *Planisphaerium*, see Heiberg.

RE = Pauly-Wissowa-Kroll, *Real-Encyclopädie der classischen Altertumswissenschaft*.

Ṣāʿid al-Andalusī, *Ṭabaqāt al-umam*, ed. Ḥ. Bū ʿAlwān, Beirut 1985.

Samsó, J., 'Maslama al-Majrīṭī and the Alphonsine Book on the Construction of the Astrolabe', *Journal for the History of Arabic Science* 4 (1980), 3-8.

Sezgin, F., *Geschichte des arabischen Schrifttums*, Iff., Leiden 1967ff.

Sinisgalli, R., and S. Vastola, *Il planisfero di Tolomeo*, Florence 1992 (Collection Domus Perspectivae, I).

Suter, H., A. Björnbo and R. Besthorn (ed.), *Die astronomischen Tafeln des Muḥammed ibn Mūsā al-Khwārizmī*, Copenhagen 1914.

Thomson, R. B. (ed.), *Jordanus de Nemore and the Mathematics of Astrolabes:* De plana spera, Toronto 1978.

Toomer, G. J., article 'Ptolemy', *DSB* XI (1975), 186ff.

Vajda, G., 'Quelques notes sur le fonds de manuscrits arabes de la Bibliothèque Nationale de Paris', *Rivista degli studi orientali* 25 (1950), 1-10.

Vernet, J., and M. A. Catalá, 'Las obras matematicas de Maslama de Madrid', *Al-Andalus* 30 (1965), 15-45.

van der Waerden, B. L., article 'Klaudios Ptolemaios' (works: *Planisphaerium*), *RE* 23,2 [46. Halbband], Stuttgart 1955, col. 1829-31.